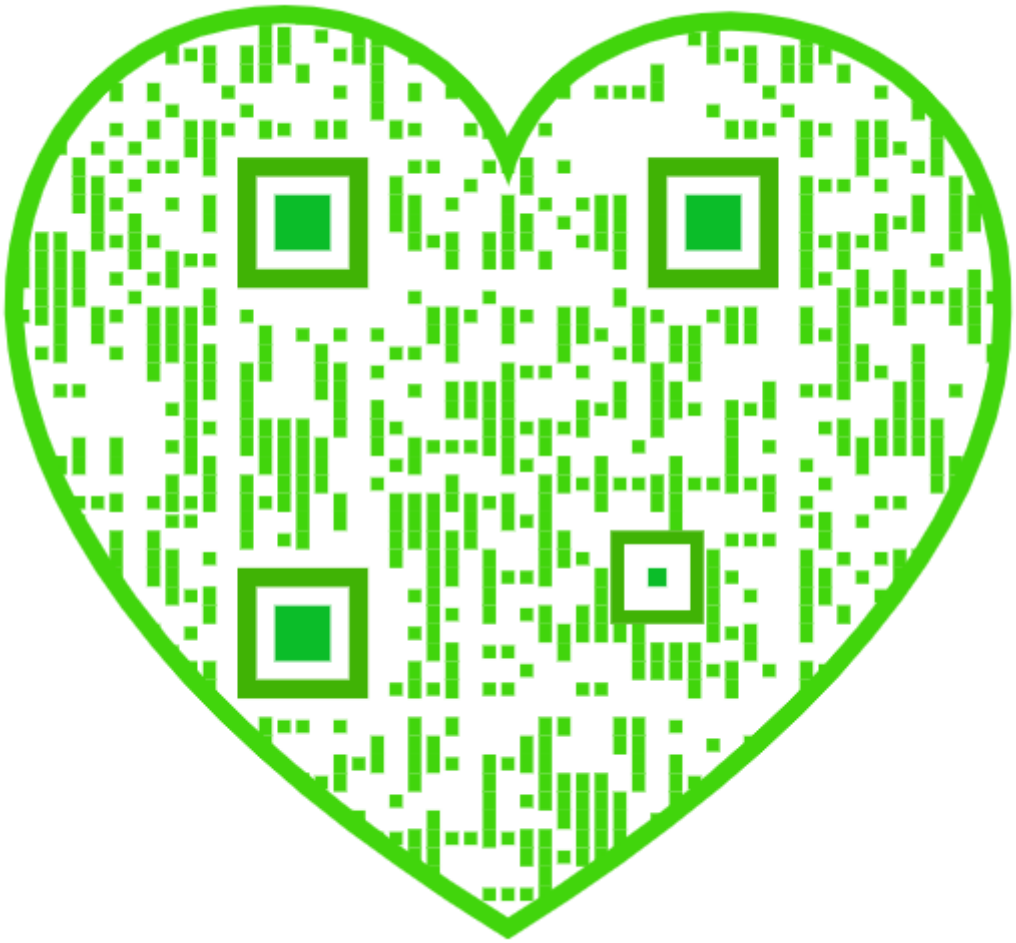

Electric Transmission

HAND-BOOK,

WITH TWENTY-TWO ILLUSTRATIONS AND TWENTY-SEVEN TABLES.

BY

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PREFACE.

This little book belongs to the Hand-Book series, which has been received with favor during the last few years. The success which attended the publication of these unpretentious volumes and the large number of inquiries from owners of water and steam power and other capitalists, as well as from agents of hydraulic and electric power companies, engineers of power transmission stations, motor inspectors and other persons interested in power transmission, induced the author to prepare this new hand-book. As usual the author has endeavored to avoid as much as possible the use of scientific terms and has employed only simple algebraic formulæ. For those who wish to study the subject of electric power transmission beyond the compass of this hand-book the following named works are recommended: "Electric Transmission of Energy," by Gisbert Kapp, C. E.; "Kritische Vergleichung der Kraftuebertragung mit den Gebrauechlichsten Mechanischen Uebertragungs Systemen," by A. Berinser; "Dynamo Electric Machinery," by Prof. S. P. Thompson; 'Electric Motors," by F. J. Sprague (late Ensign U. S. N.), a paper read before U. S. Naval Institute, Annapolis, May 16, 1887; "The Transmission of Power by Electricity," by F. J. Sprague, a lecture delivered before the Franklin Institute, Nov. 12, 1888; "Some Applications of Electric Transmission," by F. J. Sprague, a lecture delivered before the students of Sibley College and published in the *Scientific American*, July 20 and 27 and Aug. 3, 1889; the papers of George W. Mansfield, Richard P. Rothwell, Francis A. Pocock, H. C. Spaulding

and others, read before the American Institute of Mining Engineers, and the paper of H. Ward Leonard, read before the Association of Mining Engineers of the Province of Quebec, April 29, 1891. This work was almost ready for the press, when Kapp's Cantor Lectures and C. E. L. Brown's paper before the Electrical Society of Frankfort-on-the-Main were received.

Reference has been made, as far as time and space would permit, to ideas found in these works.

F. B. BADT.

Chicago, June, 1891.

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INTRODUCTORY.

The most prominent modes of long distance transmission, in use to-day, are: The steam, hydraulic, wire rope, pneumatic and electric systems.

For distances over one-half mile or where power must be distributed to a large number of smaller machines, the electric system has been proven to be by far the most economical.

Steam can be transmitted economically for short distances only, owing to the large loss in condensation in long steam pipe lines, which are very expensive, both as to first cost and repairs; besides fuel is easily portable and a number of isolated steam plants in most instances can be operated cheaper than a large boiler plant, distributing steam through many thousand feet of steam pipe. If fuel is scarce and transportation high, the use of steam power, of course, will be unprofitable in either case.

Water power is portable only to a very limited extent. Mountains or steep grades may necessitate tunneling or the construction of very expensive canals. In the past, manufacturers were, therefore, compelled to erect their factories, mills, etc., close to the river or falls instead of transmitting the water power to locations best adapted for their purposes. In many instances water power could not be utilized at all; for example in mining, as, of course, the location of a mine could not be changed.

Wire rope transmission cannot be applied for long distances nor can it transmit great amounts of power. The loss by friction and the cost of maintenance of heavy cables are too great. It is claimed for the cable car systems in Chicago (which may be considered as an illustration of wire rope transmission) that their commercial efficiency is less than 20 per cent.; or, in other words, 80 per cent. of the engine power is wasted in keeping the cable in motion while only 20 per cent. is delivered at the grip cars. No wonder electric street cars are taking the lead. They will eventually replace the cable system, except in a few instances

as pointed out in Theo. P. Bailey's paper read before the Chicago Electric club, November 17, 1890.

Compressed air was until a few years ago the only method of transmitting power to great distances. In mining it was used almost exclusively. This power was used in the construction of the famous St. Gothard tunnel which connects the Italian railway systems with those of Switzerland and Germany. If such a tunnel should be built to-day, it may be safely said that electric energy would be the agency. The main sources of water power in Switzerland are, as in the mountainous western states, characterized by small volume and great head, and could easily be used for driving water wheels of the Pelton or similar types coupled directly to electric generators.

Probably the largest pneumatic transmission plant in this country is near Marquette, Mich., which delivers at a distance of three miles 390 horse power as indicated at the engines at the Chapin and Ludington mines This 390 horse power represents only 27 per cent. of the horse power indicated by the compressors at the power station. The cost of this installation was about $500,000, the pipe line alone costing over $60,000. An electric transmission plant to deliver 390 horse power could be erected for about one sixth the cost of the pneumatic plant.

These few notes will give an approximate idea of the commercial value of the systems mentioned for transmitting power from one point to another beyond one-half mile distance But even for shorter distances where power must be distributed to a large number of smaller machines, the electric system is preferable. Instead of going into any lengthy arguments, a few paragraphs may be quoted from a letter dated Rock Island Arsenal, February 17. 1890, from J. M. Whittemore, colonel of ordnance commanding, to the chief of ordnance:

"The water wheels being connected in sets of four with generators, each generator will be able to transmit its electrical energy through a copper wire to any one motor at any one time at any one shop, at a maximum loss of 60 horse power out of 200, giving 140 effective horse power to do the work.

"With the wire rope and rigid shafting transmission, this 200 horse power would not suffice even to move the shaft for doing any work in shops A and B.

"With a capacity of 700 horse power, which is all that would be available under the present condition of the Water Power Pool, 510 horse power would be lost in friction and 190 left to do work.

"In electrical transmission 210 horse power would be lost, leaving 490 horse power available for work; or, a surplus of 300 horse power in favor of electricity.

"So far as the practicability of this system of transmission by electricity is concerned, there are reliable firms ready to contract for the work, and guarantee its success."

Much more might be said comparing the different systems of transmission, but as the space of this book is limited, we must refer the reader to the study of the excellent books and papers mentioned in the preface.

The main object of this little book is, first, to answer in a plain, and, we hope, practical manner such questions as power-producers and other prospective customers will ask electric manufacturers relating to power transmission; and secondly, to give practical rules and hints to engineers and motor inspectors in charge of transmission plants.

Questions often asked are:

Is electric power transmission a practical and commercial success and not a mere experiment?

What are the general advantages of electrical transmission over other systems?

To what distances may electric energy be transmitted, how many mechanical horse power can you recover at the motor shaft out of one hundred mechanical horse power delivered at the generator pulleys, and what percentage of electrical energy is lost in the conductors?

What is the approximate cost of generators and conductors to deliver power at various distances?

CHAPTER I.

Statistics.

The first question can easily be answered by the following statistics:

On January 1, 1891, there were in use in the United States as nearly as can be estimated from statistics published since August, 1890:

In Central Stations—

	Horse Power.	Capital Invested.
Arc and Incandescent Lighting,	390 000	$128,000,000
Electric Railways	106,000	50,000,000
Stationary Motors on Independent Power Circuits	26,000	9,000,000
Total	522,000	$187,000,000

Isolated arc and incandescent plants are not included in the above figures

Over half a million horse power is, therefore, generated every day throughout the United States to deliver electric energy over circuits from a few hundred feet to fifteen miles and more in length. Whether this electric energy is used for street cars, for stationary motors or for lighting, is immaterial; it is electric power transmission, pure and simple. Arc or incandescent lamps may be taken out of the circuit and electric motors substituted, and, *vice versa*, arc or incandescent lamps may be operated on any power circuit. It is generally known that electric street cars are lighted and heated by the electric current which is taken from the wire conveying electric energy to the street car motors.

It is hardly necessary to say any more relating to the first question, whether electric power transmission is an experiment, considering the fact that over half a million horse power is generated every day, and almost $200,000,000 invested for this purpose in the United States.

It has already been pointed out in a general way why electric long distance transmission is superior to other modes.

The electric transmission system is very flexible—it means in most instances an air line from power to motor; the conducting wires are cold and clean, they may be bent around sharp corners, and to the casual observer, there is no evidence that they are conveying, possibly, hundreds of horse power Mining Engineer Browne of World's Fair Mine fame, who read a paper before the Chicago Electric club December 1 1890,* explained most graphically the advantages of electricity in mines, and the drawbacks in long compressed air or steam pipe lines.

Steam especially is disliked by miners as it heats the mines, fills the air with moisture, and makes the problem of ventilation still more difficult.

New applicat ons of electric transmission of power are made almost daily, as is shown in the following quotation from the *Mining and Scientific Press* of San Francisco:

"In this day of steam electricity and fast transportation in general, the motive power of the old canal – the mule—can not be considered as a thing to be continued. In discussing the question now being agitated in Eastern Pennsylvania and New York in regard to the necessity for

* See *Western Electrician*, Dec. 6, 1890.

remodeling the motive power employed by the Washington & Cumberland Canal company, in view of the rapidly increasing coal and timber business crowding upon their lines of transportation, the *American Manufacturer* says: 'The motive power of the old canal—the mule—is not to be thought of. Steam offers but few more advantages, because it takes up so great a percentage of the carrying capacity of the boat, while the locks and the amount of help required stand in the way of using tugs. It was finally suggested that electricity should be used. The trolley system, by which small screw propellors could be run by electricity, was looked upon as the best solution of the problem. By using electricity the weight of the machinery to be carried would be small, and one man could control the motor and the steering apparatus without trouble. The plan is under serious consideration, and will probably be given a thorough test. If it prove successful it may result in the restoration of abandoned canals and induce capital to seek an investment in new ones between points where heavy freights not requiring rapid transit, are shipped extensively.'"

Another application of electric transmission is that for irrigation. W. Forman Collins pointed out some time ago how vast areas now barren and worthless can, by this means, be made fertile and enormously enhanced in value. Mr. Collins refers in this connection to the report of Col. W. Tweeddale to the Kansas State Board of Agriculture.

It will be only a short time when the old dream of utilizing Niagara Falls for electric transmission will be a reality.

Before the application of electricity these power transmission schemes were never thought of. Only electricity enables man to transmit natural water powers economically to the places of application.

CHAPTER II.

General Data on Electric Transmission.

The three elements of electrical transmission of power are: (1st) The generators which are placed at the power station and which are driven by the water wheel or steam engine or other prime mover; (2d) the copper conductors which are placed on poles like telegraph wires, and

which conduct the electric current from the generators to (3d) the motors, which deliver the electrical energy to all kinds of machinery. The motors are either belted or geared to these machines.

Electric generators are machines for converting mechanical into electrical energy, while electric motors reconvert electrical into mechanical energy.

The capacity of generators is therefore given by the manufacturers in watts or in electrical horse power while the capacity of motors is generally given in mechanical horse power.

Hence, in order to test these machines, it is necessary to have electrical instruments (ampere meter and voltmeter) to measure the output of the generator in electrical horse power, while the output of the motor should be measured in *mechanical* horse power by means of a dynamometer.*

In practice, however, engineers often measure the electrical horse power delivered at the motor terminals and then deduct a certain percentage to get the rating in mechanical horse power.

Ordinarily electric manufacturers allow for motors up to 20 horse power, 1,000 watts per mechanical horse power, indicating 75 per cent efficiency of the motor; from 20 to 50 horse power, 900 watts per mechanical horse power, indicating 83 per cent. efficiency of the motor; over 50 horse power, 830 watts per mechanical horse power, indicating 90 per cent. efficiency of the motor.

A similar rule will hold good for generators. Up to 20 horse power the output in electrical horse power will be about 75 per cent. of the mechanical horse-power applied to the pulley. From 21 to 50 horse power the output in electrical horse power will be about 83 per cent. of the mechanical horse power applied to the pulley. Over 50 horse power the output in electrical horse power will be about 90 per cent. of the mechanical horse power applied to the pulley.

Seven hundred and forty six watts (one watt = ampere × volt) equal one electrical horse power. If we therefore deliver as in the example mentioned, 1000 watts—1.333 electrical horse power—to the motor, and the latter has a commercial efficiency of 75 per cent., it will deliver 75 per cent. of 1.333 = one mechanical horse power at its pulley.

Or, if we deliver one mechanical horse power to the pul-

*The cradle dynamometer of Prof. Brackett and the floating or hydrostatic dynamometer of Prof. Webb are perfectly applicable to tests of motors for efficiency.

ley of a generator of 90 per cent. efficiency, the electrical output will be 90 per cent. of one electrical horse-power, or .9 electrical horse power, or .9×746=671 4 watts.

It is only fair to state that machines of the best makes will show higher efficiences, but these figures may be considered as a conservative est mate.

In the following there will be assumed both for generator and motor a commercial efficiency of 90 per cent. or a loss of 10 per cent. in conversion.

This is fair, for the reason that in most cases of long distance transmission large units will be used.

By placing the generator and motor near each other, assuming no loss in the connecting wires, we get—

One hundred per cent. mechanical energy delivered at generator pulley	100
Loss by conversion in dynamo 10 per cent	10
	90
Loss by reconversion in motor, 10 per cent. of 90	9
	81

This shows that out of 100 mechanical horse power applied to the generator pulley, 81 mechanical horse power shou d be recovered at the motor shaft if loss in the conductors could be avoided.

This efficiency of a couple of electric machines connected as generator and motor with practically no loss in the connecting conductors is often called the "*Couple Efficiency.*"

In practice the generator and motor are so far apart that there is loss of electrical energy in overcoming the resistance of the conductors This loss depends upon three factors, viz.: Distance between generators and motors, electric pressure at generators and size of copper conductors. For a given case the first factor, distance, is constant; pressure and size of conductors are variable and may be determined at will; therefore, the loss in the conductors may be any percentage desired.

It should be stated that only "compiete metallic circuits" will be considered, or, in other words, it will be assumed that the generator is connected to the motor by means of two conductors. "Earth returns," which are mainly used in electric railway work, will not be considered.

If a "couple efficiency" of 81 per cent. and a loss of say 10 per cent. in the conductors is assumed, there will be:

Couple efficiency 81.0
Loss in the wire, 10 per cent. of 81 ..... 8.1

72.9

Or the commercial efficiency of the transmission system from generator pulley to motor shaft would be 72.9 or almost 73 per cent.

CHAPTER III.

Commercial Efficiency of Transmission Plants.

Table I shows the relation of the different factors of electrical transmission to each other, assuming an efficiency of generators and motors of 90 per cent (or a couple efficiency of 81 per cent.) and losses in the conductors varying from 0 per cent. to 50 per cent.

Many questions which come up in electric power transmission may be answered by simply looking at this table.

EXAMPLE 1.—137 horse power is wanted at the motor pulley, the commercial efficiency of the electric system to be about 68 per cent. What is the loss of electrical energy or the "drop" in the wire; what must be the capacity of the generator in electrical horse power, and how many mechanical horse power must be delivered at the generator pulley?

Solution.—We select in column 6 the nearest number to 68, viz, 68.85, and find on the same line in the third column 15 per cent., which is the drop in the conductor. We find in column 4 the figure 1.3072 which, multiplied by 137, gives us the electrical horse power of the generator, and in column 5 we find 1.4524, which, multiplied by 137, gives the mechanical horse power required to drive the generator.

Hence electrical horse power of generator = 179 09.

Mechanical horse power required to drive generator= 198.98.

EXAMPLE 2.—There is a water power available which is estimated at the falls at 580 horse power. Our customer wishes to transmit at least 70 per cent. of this power. What is the power required at the water wheels, what capacity of generator how much drop in the wire, and what is the commercial efficiency of the *electric* system?

Solution.—If we assume the commercial efficiency of a good turbine wheel at 80 per cent. we can deliver at the

EFFICIENCY IN ELECTRIC POWER TRANSMISSION.

TABLE I.

1. Mech. H. P. required at motor shaft. N.	2. El. H. P. to be transmitted to motor.	3. Per cent. loss in conductor. x.	4. El. H. P. required in generator.	5. Mech. H. P. to be delivered at generator pulley.	6. Efficiency of whole system in per cent. l.
1.00	1.1111	0.0	1.1111	1.2346	81.00
1 00	1.1111	1.0	1.1223	1.2470	80.19
1.00	1.1111	2.0	1.1337	1.2597	79.38
1.00	1.1111	3.0	1.1454	1.2727	78.57
1.00	1.1111	4.0	1.1574	1.2860	77.76
1.00	1.1111	5.0	1.1696	1.2995	76.95
1.00	1.1111	6 0	1.1721	1.3134	76.14
1.00	1.1111	7.0	1.1947	1.3275	75.33
1 00	1.1111	8.0	1.2077	1.3419	74.52
1.00	1.1111	9.0	1.2210	1.3567	73.71
1.00	1.1111	10.0	1.2345	1.3717	72 90
1.00	1.1111	12.5	1.2698	1.4109	70.88
1.00	1.1111	15.0	1.3072	1.4524	68.85
1.00	1.1111	17.5	1.3468	1.4964	66.83
1.00	1.1111	20.0	1.3888	1.5447	64.80
1.00	1.1111	22.5	1.4336	1.5929	62.78
1.00	1.1111	25.0	1.4815	1.6461	60.75
1.00	1.1111	27.5	1.5325	1.7028	58.73
1.00	1.1111	30.0	1.5873	1.7636	56.70
1.00	1.1111	32.5	1.6464	1.8293	54.68
1.00	1.1111	35.0	1.7094	1 8993	52 65
1.00	1·1111	37.5	1.7778	8.9753	50.63
1 00	1.1111	38.3	1.8000	2 0000	50.00
1.00	1.1111	40.0	1.8518	2.0576	48.60
1.00	1.1111	42.5	1.9323	2.1470	46.58
1.00	1 1111	45.0	2.0201	2.2446	44.55
1.00	1.1111	47.5	2.1164	2.3515	42.53
1.00	1.1111	50.0	2.2222	2.4623	40.50

generator pulley 580×.80=464 horse power. The prospective customer wishes, however, to recover at the motors 70 per cent. of 580 horse power, which is 580×.70=406 horse power; 406 horse power is $\frac{406}{464}$ = 87.5 per cent. of 464 horse power; or in other words the electric transmission system should have a commercial efficiency of 87.5 per cent., of course, is impossible, as the highest efficiency obtainable—the couple efficiency—is only 81 per cent. Still customers will ask impossibilities, and often agents of electric companies will promise them.

Table I, therefore, will solve not only possible problems, but show at once certain impossibilities.

CHAPTER IV.

Inter-relation of Electromotive Force, Current, Distance, Cross-Section and Weight of Conductor.

In the foregoing considerations the electrical pressure of the motors and the distance between generator and motor were not made elements. Depending upon the electrical pressure or electromotive force and the distance are, however, for a given number of horse power to be transmitted, the current, the cross-section and the weight of the conductor.

The inter-relation between these factors may be best expressed in the following rules:*

RULES FOR THE INTER RELATION OF ELECTROMOTIVE FORCE, CURRENT, DISTANCE, CROSS-SECTION AND WEIGHT OF COPPER CONDUCTOR.

With any amount of energy transmitted, the electromotive force and the current will vary inversely.

With any given work done, loss on the line, electromotive force at the terminals of the motor and distribution, the weight of the copper will vary as the square of the distance, its cross section of course varying directly as the distance.

With the same conditions, the weight will vary inversely as the square of the electromotive force used at the motor.

*The rules printed in italics are taken from Frank J. Sprague's papers mentioned in the preface.

With the same cross-section of conductor, the distance over which a given amount of power can be transmitted will vary as the square of the electromotive force.

If the weight of the copper is fixed, with any given amount of power transmitted and given loss in distribution the distance over which the power can be transmitted will vary directly as the electromotive force.

These rules will be further explained in the following considerations:

Electric power transmission means transference of energy which may be measured in units of quantity and pressure.

One electrical horse power equals 746 watts, one watt being the product of one ampere by one volt.

746 watts (1 electrical horse power) may now be obtained by the product of 1 ampere and 746 volts or by 746 amperes and 1 volt, or by X amperes $\times \frac{746}{X}$ volts.

Hence with any given amount of energy the electromotive force and the current will vary inversely.

In our demonstrations and formulæ we shall use the following symbols, viz.:

E (or E. M. F.)=Electromotive force at terminals of motor in volts.

V=Number of volts lost in conductor.

E+V=Electromotive force at terminals of generator in volts.

C=Current in amperes required for N mechanical horse power delivered by motor shaft.

D=Distance in feet (plus 5 per cent. for sag) of current transmitted one way. (Distance equals one-half length of complete metallic circuit.)

N=Number of mechanical horse power delivered by motor shaft.

M (or CM or d^2)=Area of cross-section of wire in circular mils.

R_1=Line resistance in ohms of complete metallic circuit.

Wt=Weight in pounds of complete metallic circuit.

a=Commercial efficiency of motor.
b=Commercial efficiency of generator.
h=Commercial efficiency of whole electric system.
%=Per cent. of electrical energy lost in the conductor.

Written as a decimal fraction; for instance 90%=.90.

As one electrical horse power equals 746 watts, the current for one horse power equals $\frac{746}{E}$

In order to deliver N mechanical horse power at the motor shaft $\frac{N}{a}$ electrical horse power must be delivered at the motor terminals.

Hence for N mechanical horse power delivered by the motor $C=\frac{746\times N}{E\times a}$ (Formula 1)

The resistance of the conductor, however, is twice the distance multiplied by the resistance of one mil-foot of copper divided by the number of circular mils, or assuming the resistance of one mil-foot of commercial copper = 10 74 ohms* we get the formula:

$$R_1=\frac{2D\times 10.74}{M}$$

or $R_1=\frac{21.48\times D}{M}$ (Formula 2.)

The "drop" in the line, of course, equals resistance of line multiplied by current or $V=\frac{C\times 21.48\times D}{M}$

Transforming this formula we get:

$M=\frac{C\times 21.48\times D}{V}$ (Formula 3.)

Substituting the value for $C=\frac{746\times N}{E\times a}$, we get

$M=\frac{16,024\times N\times D}{E\times a\times V}$; to facilitate calculations we substitute the approximate value, 16,000, for 16,024, and have the general formula:

$M=\frac{16000\times N\times D}{E\times a\times V}$ (Formula 4.)

which is the important formula for calculating the cross-section of the conductor.

EXAMPLE.—The motor has 90% efficiency; the electromotive force of motor = 500 volts. We must deliver 50 horse power at the motor shaft at a distance of 6,000 feet from the generator at 10% loss in the conductors. What is the size of the conductor?

*True ohms. Compare notes on page 93.

Solution.—The electromotive force of the generator is the efficiency of the circuit .90 divided into the electromotive force of the motor 500—555 5. or as a round figure 556 volts; the drop in the line therefore is 556 minus 500—56 volts.

Substituting these values in formula 4

$$M=\frac{16000\times N\times D}{E\times a\times V}$$ we get:

$$M=\frac{16000\times 50\times 6000}{500\times 90\times 56}=190476,$$

which is a wire a little larger than 000 B. W. G. (See Chapter XIV on gauges.)

The above formula also proves that with any given work done, given loss on the line and electromotive force of motor, the number of circular mils of the conductors will vary directly as the distance. Hence with given conditions, if we double the distance we must also double the cross section, or if we treble the distance we must treble the cross-section.

The weight of a foot of the conductor of course increases also in direct proportion to its cross-section. If we therefore double both cross-section and distance the total weight of the conductor will be increased four-fold, or if we treble both cross-section and distance, the total weight of the conductor will be increased nine fold.

This shows that with the conditions given, the weight of the copper will vary as the square of the distance.

This may also be proven in the following way:

We found for the size of wire the formula

$$M=\frac{16000\times N\times D}{E\times a\times V}$$

If y equals the weight of one mil-foot of copper, then the weight *per foot* of the conductor would be

$$\frac{16000\times N\times D\times y}{E\times a\times V}$$ and the total weight for 2 D, which is the total length of the circuit, would be

$$\frac{16\ 00\times N\times D\times y\times 2D}{E\times a\times V}$$

Substituting the value for y=.000003027:49 we get: Weight of complete metallic circuit in pounds—

$$\frac{.0968688\times N\times D^2.}{E\times a\times V}$$

For easy calculation and to allow for waste in the erection of the line, we substitute the approximate value $\frac{1}{10}$ for .0968688 and get the general formula:

$$\text{Wt} = \frac{N \times D^2}{10 \times E \times a \times V} \qquad \text{(Formula 5)}$$

If we now assume K as the cost of bare copper wire per pound in cents $\frac{K}{100}$ will be the cost of copper per pound in dollars, hence:

$$\text{Cost in \$} = \frac{N \times D^2}{10 \times E \times a \times V} \times \frac{K}{100}$$

$$\$ = \frac{N \times D^2 \times K}{1000 \times E \times a \times V} \qquad \text{(Formula 6.)}$$

EXAMPLE. — D=15,000 feet.
N=100 horse power.
E=500 volts.
a=.90 (90% efficiency).
V=80 volts.
K=20 cents.

Solution.—Cost of bare copper wire=

$$\frac{100 \times 5000 \times 5000 \times 20}{1000 \times 500 \times .90 \times 80} = \$1388.89$$

Assuming the same conditions but the efficiency of the motor as 80% instead of 90% we find the cost of the wire $1,562.50, or in other words, we must pay more for copper in order to deliver the same number of horse power by a motor of lower commercial efficiency.

Formula 6 proves also that the cost of the conductor increases in direct proportion to the horse power to be delivered at the motor shaft and to the square of the distance and decreases in inverse proportion to the electromotive force and the efficiency of the motor, and the number of volts lost in the line.

For least cost of the copper conductor it is therefore necessary to make the electromotive force and efficiency of the motor and the loss in the line as high as possible.

If we substitute the value for $N = \frac{C \times E \times a}{746}$ derived from formula 1, viz.: $C = \frac{746 \times N}{E \times a}$ in formulæ numbers 5 and 6, we get

$$\text{Wt.} = \frac{C \times D^2}{7460 \times V} \qquad \text{(Formula 7.)}$$

$$\$ = \frac{C \times D^2 \times K}{746{,}000 \times V} \qquad \text{(Formula 8.)}$$

This shows that the weight and cost of the conductor increase in direct proportion to the current. In order to get the cost of the conductor very low it is therefore necessary to reduce the current strength to a permissible minimum. As a definite amount of electrical energy depends, however, on the product of current and electromotive force; the electromotive force must be increased in the same ratio as the current is reduced, which shows the same fact, viz: For least cost of conductor make the electromotive force of the motor as high as permissible.

CHAPTER V.

Conditions of Plant for Least Operating Expenses.

We have seen in the previous chapters that a certain percentage of electrical energy must be lost in the conductors; this loss, of course, involves continuous operating expense, as the prime mover (steam, water, etc.) and the electric generator must produce an additional amount of energy which is lost in the conductors. It is a loss in a commercial sense only, as this so-called "lost" energy reappears as heat in the conductor.

This loss can be decreased and power economized by using conductors of greater cross section which, of course, would involve a greater outlay for copper. On the other hand, to reduce the first cost, we should employ conductors of the least possible cross-section. We can now easily see that *for any given case the cheapest in the long run will be a certain size of conductor for which the interest on its first cost plus annual cost of energy wasted in the conductor, becomes a minimum.*

Sir William Thomson's law states that:

The most economical area of conductor will be that for which the annual interest on capital outlay equals the annual cost of energy wasted.

We might write this in the form of an equation:

Annual cost of energy wasted = Interest on capital outlay for conductor.

The cost of one electrical horse power hour at the terminals of the generator including interest and depreciation

on the building, motive power and electric generator, multiplied by the number of horse power hours per year wasted in the conductor, must be considered "cost of energy."

The interest on capital outlay for conductor plus allowance for repairs and depreciation taken for the year gives the other side of the equation.

Both sides of the equation added together give the annual cost of transmitting the electrical energy.

Gisbert Kapp remarks very pertinently in this connection:

"It should be remembered that this law in the form here given only applies to cases where the capital outlay is strictly proportional to the weight of metal contained in the conductor In practice this is however, seldom correct. If we have an underground cable, the cost of digging the trench and filling in again will be the same, whether the cross-sectional area of the cable be one-tenth of a square inch or one square inch: and other items, such as insulating material, are if not quite independent of the area, at least dependent in a lesser degree than assumed in the formula. In an overhead line we may vary the thickness of the wire within fairly wide limits without having to alter the number of supports, and thus there is here also a certain portion of the capital outlay which does not depend on the area of the conductor.

"*Hence we should state more correctly that the most economical area of conductor is that for which the annual cost of energy wasted is equal to the annual interest on that portion of the capital outlay which can be considered to be proportional to the weight of metal used.*

"Prof. George Forbes, in his Cantor lectures on 'The Distribution of Electricity,' delivered at the Society of Arts, in 1885, called that portion of the capital outlay which is proportional to the weight of metal used, '*The Cost of Laying One Additional Ton of Copper*,' and he showed that for a given rate of interest inclusive of depreciation, and a given cost of copper, *the most economical section of the conductor is independent of the electromotive force and of the distance, and is proportional to the current.*

"Having in a given system of electric transmission settled what current is to be used, we can, by the aid of Sir William Thomson's law, proceed to determine the most economical size of conductor To do this we must know the annual cost of an electrical horse power inclusive of interest and depreciation on the building, prime mover and

dynamo; we must know what is the cost of laying one additional ton of copper, and we must settle in our mind what interest and depreciation shall be charged to the line. These points will serve to determine the constants of our formulæ, and then the calculation can easily be made."

In order to facilitate these computations, Prof Forbes published some tables. Similar tables calculated by Prof. H. S. Carhart on the basis of dollars instead of pounds sterling will be found on pages 18 and 19.

These tables have been calculated in such a way that when the investigator has decided upon the proper allowance to be made for cost of laying one additional ton of copper under the conditions of his particular plant, the percentage of allowance for interest, etc., he can then determine at once the proper size of conductors to employ

Thus in Table II he follows the columns headed with the assigned cost of conductors until he reaches the line corresponding to percentage allowed for interest, etc., and there finds a number. With this number he turns to Table III, and starting at the left, on the line marked with the number expressing the cost of one electrical horse power per annum, he follows along to the right till he comes to the number nearest the one taken from Table II. The number standing at the head of the column in which he finds this exact or nearest approximate number is the sectional area of the conductor, in square inches or circular mils required to carry 100 amperes with maximum economy under the conditions assumed.

Thus, in Table II, suppose we assume $900 as the cost of laying one additional ton of copper and 12 as the rate of interest to be allowed. Then we find the number 216 at the designated intersection. Turning to Table III, let 50 amperes be the current to be transmitted, and $75 the estimated cost of one horse power per annum; then following along this line we come to 219 as the number nearest to 216. At the head of the column in which 219 is, we find .20 square inches or 254,640 circular mils as the sectional area of the conductor, required to carry 100 amperes. But since the current is 50 and not 100 amperes the size of the conductor should be

$$\frac{254\ 640 \times 50}{100} = 127,320 \text{ circular mils.}$$

If a larger conductor should be selected the interest on the capital invested for copper would become too great, and if a smaller conductor should be decided upon the operating

TABLE II.

COST OF LAYING ONE ADDITIONAL TON OF COPPER.

Annual Allowance for Interest and depreciation in %	$300	$325	$350	$375	$400	$425	$450	$475	$500	$550	$600	$650	$700	$750	$800	$900	$1000	$1100	$1200	$1400	$1600	$1800	$2000
5	030	033	035	038	040	043	045	048	050	055	060	065	070	075	080	090	100	110	120	140	160	180	200
6	036	039	042	045	048	051	054	057	060	066	072	078	084	090	096	108	120	132	144	168	192	216	240
7	042	046	049	053	056	060	063	067	070	077	084	091	098	105	112	126	140	154	168	196	224	252	280
8	048	052	056	060	064	068	072	076	080	088	096	104	112	120	128	144	160	176	192	224	256	288	320
9	054	059	063	068	072	077	081	086	090	099	108	117	126	135	144	162	180	198	216	252	288	324	360
10	060	065	070	075	080	085	090	095	100	110	120	130	140	150	160	180	200	220	240	280	320	360	400
12	072	078	084	090	096	102	108	114	120	132	144	156	168	180	192	216	240	264	288	336	384	432	480
14	084	091	098	105	112	119	126	133	140	154	168	182	196	210	224	252	280	308	336	392	448	504	560
16	096	104	112	120	128	136	144	152	160	176	192	208	224	240	256	288	320	352	384	448	512	576	640
18	108	117	126	135	144	153	162	171	180	198	216	234	252	270	288	324	360	396	432	504	576	648	720
20	120	130	140	150	160	170	180	190	200	220	240	260	280	300	320	360	400	440	480	560	640	720	800
25	150	163	175	188	200	213	225	238	250	275	300	325	350	375	400	450	500	550	600	700	800	900	1000

TABLE III.

SECTIONAL AREA FOR 100 AMPERES IN SQ. INCHES AND CIRCULAR MILS.

Annual cost of one electrical horse power at generator terminals. (Inclusive of interest and depreciation on buildings, motive power and generator).	Circular Mils.	127,320	140,052	152,784	165,516	178,248	190,980	203,712	216,444	229,176	241,908	254,640	267,372	280,104	292,836	305,568	318,300	331,032	343,764	356,496	369,228	381,960	394,692	407,424	420,156	432,888	445,620
	Sq. Ins.	.10	.11	.12	.13	.14	.15	.16	.17	.18	.19	.20	.21	.22	.23	.24	.25	.26	.27	.28	.29	.30	.31	.32	.33	.34	.35
	$ 25	291	240	202	172	148	129	114	101	090	081	073	066	060	055	051	047	043	040	037	035	032	...	...	...	...	...
	30	349	289	242	207	178	155	136	121	108	097	087	079	072	066	061	056	052	048	045	042	039	036	...	...	...	...
	35	407	337	283	241	208	181	159	141	126	113	102	092	084	077	071	065	060	056	052	048	045	042	040	...	...	...
	40	465	385	323	275	238	207	182	161	144	129	116	105	096	088	081	074	069	064	059	055	052	048	045	043	...	...
	45	524	433	364	310	267	233	204	181	162	145	131	118	108	099	091	084	077	072	067	062	058	054	051	048	045	...
	50	582	481	404	344	297	259	227	201	180	161	146	132	120	110	101	093	086	080	074	069	065	061	057	053	050	048
	55	640	529	445	379	327	285	250	221	198	177	160	145	132	121	111	103	095	088	082	076	071	067	063	059	055	052
	60	698	577	485	413	356	310	273	241	216	193	175	158	144	132	121	112	103	096	089	083	076	073	068	064	060	057
	65	757	625	526	448	386	336	295	261	234	209	190	171	156	143	131	121	112	104	097	090	084	079	074	069	065	062
	70	815	673	566	482	416	362	318	281	252	225	204	185	168	154	141	131	120	112	104	097	091	085	080	075	070	067
	75	873	721	606	517	445	388	341	302	270	241	219	198	180	165	152	140	129	120	111	104	097	091	085	080	076	071
	80	931	769	647	551	475	414	364	322	287	257	233	211	192	176	162	149	138	128	119	111	103	097	091	086	081	076
	85	989	817	687	585	505	440	386	342	305	274	248	224	204	187	172	158	146	136	126	118	110	103	097	091	086	081
	90	1047	865	727	620	534	466	409	362	323	290	262	237	216	198	182	167	155	144	134	125	116	109	102	096	091	086
	95	....	914	768	654	564	491	432	383	341	306	277	251	228	209	192	177	164	152	141	131	123	115	108	102	096	090
	100	....	....	808	689	594	517	455	403	359	322	291	264	240	220	202	186	172	160	148	138	129	121	114	107	101	095
	105	....	....	....	723	624	543	477	423	377	339	306	277	252	231	212	195	181	168	156	145	136	127	119	112	1 6	100
	110	....	....	....	....	653	569	500	443	395	355	320	290	264	242	222	204	189	176	163	152	142	133	125	118	111	105
	115	..	....	....	....	....	595	523	463	413	371	335	304	276	253	232	214	198	184	171	159	149	139	131	123	116	109
	120	....	....	....	....	....	....	546	483	431	387	349	317	288	264	242	223	207	192	178	166	155	145	136	128	121	114
	125	....	....	....	....	....	....	....	503	449	403	364	330	301	275	253	233	215	200	186	173	162	151	142	134	126	119
	130	....	....	....	....	....	....	....	....	467	419	378	343	313	286	263	242	224	208	193	180	168	157	148	139	131	124

expenses (annual cost of one electrical horse power at generator terminals inclusive of interest and depreciation on building, motive power and generator) would become too great.

The installation as determined by Thomson's law will be the cheapest in the long run.

We may again quote Kapp:

"We have calculated the area of our conductor under the supposition that the maximum current of 50 amperes would be flowing during all the hours per annum that the installation is at work. In other words, we have assumed that the motor when at work should always give full power. This will, in practice, seldom be the case. Whether we want the current for propelling railway cars, or producing the electric light, or working lathes and other tools generally, or giving power for a whole mill, the amount of energy required at various times will be different It has been shown that energy can be transmitted in either of three ways. First, by keeping the current constant and varying the electromotive force of the generator in accordance with the demand for power at the receiving station. Secondly, by keeping the electromotive force constant, and varying the current in accordance with the demand for power. Thirdly, by varying both current and electromotive force. In the first case, where the current is constant, the above formula for the most economical area of conductor is at once applicable whatever may be the difference in the energy transmitted at various times of the day or year. In the two other cases, however, a correction must be applied to the formula in order that account may be taken of those hours when a reduced current is passing, and when the most economical area of conductor would be smaller than that corresponding to the full current and maximum energy transmitted. This correction must evidently be applied in this form: We make our calculation not for the full current but for the reduced current; the greater the reduction the greater the number of hours during which a reduced current is passing as compared to the number of hours during which the full current is passing. At first sight it might seem as if this reduced or mean current could be determined by simply dividing the total number of ampere hours per annum by the number of hours per annum This, however, would not be correct, for the reason that the energy wasted varies not with the current itself, but with the square of the current."

To facilitate the calculations Prof. Forbes gives a table, which we have extended and reproduced in a little different form in Table IV.

TABLE IV.

TO FIND MEAN ANNUAL CURRENT.*

Fraction of time per year during which				Ratio.	Fraction of time per year during which				Ratio.
Current. ¼	Current. ½	Current. ¾	Full Current.		Current. ¼	Current. ½	Current. ¾	Full Current.	
is passing through the conductor.					is passing through the conductor.				
0	0	0	1	1.000	¼	¼	0	½	.760
0	0	¼	¾	.944	¼	0	½	¼	.744
0	¼	0	¾	.901	½	0	0	½	729
0	0	½	½	.884	0	½	¼	¼	.718
¼	0	0	¾	.875	¼	¼	¼	¼	.685
0	¼	¼	½	.838	0	¾	0	¼	.661
0	0	¾	¼	.820	½	0	¼	¼	.650
¼	0	¼	½	.810	¼	½	0	¼	.611
0	½	0	½	.790	½	¼	0	¼	.586
0	¼	¼	¼	.771	¾	0	0	¼	.545

The figures in the columns headed: "¼ current," "½ current," "¾ current" and "Full current" represent fractions of the total annual time during which ¼, ½, ¾ of the full current and the full current is passing through the conductor.

The figures in the column headed "Ratio" are those with which the most economical area for the maximum current must be multiplied to obtain the most economical area for a varying current.

We found in the previous example that the conductor should have a sectional area of 127,320 circular mils, assuming the full current would always be flowing during running hours of the installation.

If we now assume that the plant will be in operation only eight hours during every day of the year, and that during

*NOTE.—This table was calculated from the formula:

$$\text{Mean current} = \text{current} \sqrt{\frac{(¼)\ t_1 + (½)^2 t_2 + (¾)^2 t_3 + t_4}{t_1 + t_2 + t_3 + t_4}}$$

where t_1, t_2, t_3, t_4, represent the number of hours per annum during which one-quarter, one half, three-quarters of the full current and the full current is respectively passing through the conductor (See Kapp).

four hours the full current will be delivered, during two hours ½ of the full current and during the remaining two hours only ¼ of the full current, we find in the eleventh line of the table the figures ¼, ¼, 0, ½, .760, which correspond with this case.

The proper size of the conductor will therefore be 127,320×.760 = 96,763 circular mils.

George W. Patterson, A. B., B S., Instructor in Physics, University of Michigan, has suggested an ingenious little device for getting the proper mean current to use in the application of Sir W. Thomson's law for size of conductors.

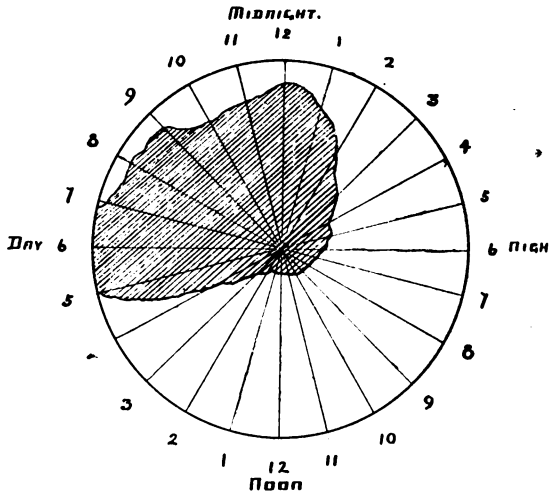


FIG. 1.—MEAN ANNUAL CURRENT.

Divide a large circle represented in Fig. 1 into 24 equal arcs, representing the 24 hours of the day, and draw the radii. Lay off from the center of the circle on the radii distances proportional to the currents at the several hours. Then draw a curve through the points thus made on the radii. Then find the area of the shaded curve in the figure by means of a planimeter. Find the radius of a circle having the same area as this shaded curve. It will be the mean current required, for each elementary area of the

shaded figure is proportional to the *square* of the corresponding radius vector.

If still greater accuracy should be desired, the circle may be divided into 48 equal arcs, representing ½ hours, or in 96 arcs, representing ¼ hours, etc.

b. Gisbert Kapp's Latest Formula.

In his third Cantor Lecture, March 2, 1891, Kapp opened his discourse by referring to certain rules laid down by Sir William Thomson, and by Professors Ayrton and Perry for calculating the best size of conductors, and it was shown that neither of these methods was universally applicable. Professors Ayrton and Perry's paper, read at the Institution of Electrical Engineers, in 1888, started with the assumption that the current has a constant value, while Sir William Thomson's rule ignores the voltage. The following table was prepared by the lecturer, and it contains all the functions entering into any system of electric transmission:

MOST ECONOMICAL CURRENT FOR ELECTRIC POWER TRANSMISSION.

D	Distance in miles.
a	Section of conductor in square inches.
E	Terminal volts at generator.
e	Terminal volts at motor.
HP_g	Brake horse power required to drive generator.
HP_m	Brake horse power obtained from motor.
c	Current in amperes.
	Efficiency of generater 90 per cent., efficiency of motor 90 per cent.
g	Cost in £ per electrical horse power output of generator.
m	Cost in £ per brake horse power output of motor including regulating gear.
$G = .9_g\ HP_g$	Cost in £ of generator.
$M = m\ HP_m$	Cost in £ of motor and regulating gear.
$t = 18.2\ D_a$	Weight in tons of copper in line.
K	Cost in £ per ton of copper, including labor in erection.
s	Cost in £ of supports of line per mile run
p	Cost in £ of one annual brake horse power absorbed by generator
q	Percentage for interest and depreciation on the whole plant.

Capital outlay, =

$$s\frac{E\,c}{746}+m\,HP_m+Ds+\frac{1.6KD^2c^2}{E\,c-830HP_m}=A.$$

Annual cost per brake horse power delivered=

$$q\frac{A}{HP_m}+f\frac{HP_g}{HP_m}$$

Put $B=\frac{E\,f}{670}+s\frac{E\,g}{746}$

$j=\frac{830}{E}HP_m$, the current which would be required if the line had no resistance,

and $\beta=j^2\frac{EB}{1.6qKD^2+EB}$,

Then the most economical current at the given voltage E is

$$c=j\left(1+\sqrt{1-\frac{\beta}{j^2}}\right)$$

$$c=j\left(1+\sqrt{\frac{1.6qKD^2}{1.6qKD^2+BE}}\right)$$

For very long distances the term under the square root approaches unity and the most economical current the value $2j$; from which it follows that under no circumstances will it be economical to lose more than half the total power in the line.

CHAPTER VI.

Conditions for Minimum Total Initial Cost of Transmission Plant.

In the previous chapter we investigated the laws governing the conditions for a transmission plant which will be the *cheapest in the long run*. In practice, however, it will be very often almost impossible to predetermine some of the factors which are necessary to employ Sir William Thomson's or Kapp's formulæ. Take, for instance, a prospective power transmission plant in our western country. Who could even approximately determine the cost of one horse power hour number of hours per annum that the maximum current, and the number of hours that three-fourths, one-half and one-quarter of this amount would be required?

These items will change rapidly in a country which is fast growing up; and while, for instance, the cost of one horse power-hour at the generator terminals may be five cents to-day, it might be only two cents in a year hence; and again, while the market for renting power may be very limited to-day, a rapidly growing mining industry, for example, may cause a great demand for power and customers would be willing to pay comparatively higher prices for current delivered every hour of the year. Nevertheless, it is true that *no system of transmission will be profitable if the cost of the power at the distant end is not less than the price which would have to be paid for its production there by water, steam, or some other agency*. Power producers investigating the advantages of electric power transmission will in most cases be satisfied to calculate the conditions for the *minimum initial cost* of the plant, leaving the exact determination of the cost of one horse power-hour and the price at which they can profitably rent power to future developments.

To a certain extent of course a plant which requires the smallest investment of money will be the cheapest to operate. As a matter of fact, the same elements which are employed in Sir William Thomson's law enter into our computations with the exception of the items: Number of annual running hours and cost of one horse power-hour at generator terminals. The whole question of minimum initial cost of plant again hinges on the proper size of the conductors.

Frank J. Sprague argues as follows: "In considering the transmission of power, some very curious and valuable facts may be demonstrated by a formula for determining the minimum cost of plant, where the amount of power at the generating station is practically not limited and where there is no line loss from leakage, the line loss being measured simply by the fall in potential.

"The cost of a plant of this character can be divided into five parts, that of the motors, the conductors, the line erection, the dynamos, and the power plant whether water or steam. I will assume that the cost of the dynamos and motors is the same per horse power or other unit, no matter what the electromotive force used may be. While this is not strictly true, for all practical purposes, with large units, and speaking from the commercial standpoint, it can be so assumed. This being the case, for any given power the cost of the motor is a constant, independent of the potential

used. With any given motor potential, the greater the loss on the line, the less the cost of the conductors, but the greater the cost of the generators. On the other hand, the less the loss on the line, the greater its cost, but the less the cost of the generating plant. *It follows, then, that the least cost to the contractor is determined when the variation in the cost of the generator is equal to that in the cost of the line.*"

Sprague develops some very interesting formulæ from which he deduces the following laws:

With fixed conditions of cost and efficiency of apparatus, the number of volts fall to get the minimum cost of the plant is a function of distance alone, and is independent of the electromotive force used at the motor.

With any fixed couple and commercial efficiency, the cost of the wire bears a definite and fixed ratio to the cost of the generating plant.

The cost of the wire varies directly with the cost of the generating plant.

If we do not limit ourselves in the electromotive force used, the cost per horse power delivered exclusive of line erection is, for least cost and for a given commercial efficiency, absolutely independent of the distance.

By the aid of these laws and Sprague's formulæ and assuming:

K = Cost in cents of *bare* copper wire per lb. delivered at the poles. ..... = 25
a = Commercial efficiency of motor. ..... = .90
b = Commercial efficiency of generator. ..... = .90
G = Cost in dollars of generator set up, per electric horse power delivered at its terminals. = 45
P = Cost in dollars of power (water) set up per mechanical horse power delivered at generator pulleys..... = 25

We have arranged some tables and diagrams which will prove extremely useful for ready reference.

We can best explain the use of these tables (V and VI) by a few examples:

EXAMPLE 1.—Distance (including 5% for sag) 7 000 feet, electromotive force at motor terminals, 500 volts. What must be the loss in the wire and electromotive force of generator for minimum cost of plant?

Solution.—We find in Table V the horizontal line 500 which represents the electromotive force of motor and the vertical 7,000 which is the distance; almost at the intersec-

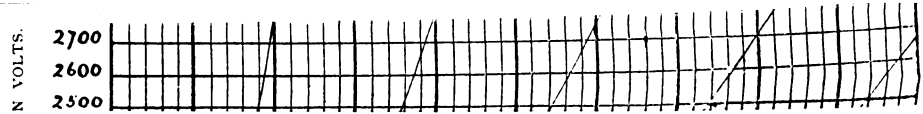


TABLE VI.

COMMERCIAL EFFICIENCY AT VARIOUS DISTANCES AND VARIOUS VOLTAGES OF MOTOR FOR MINIMUM TOTAL INITIAL COST OF TRANSMISSION PLANT.

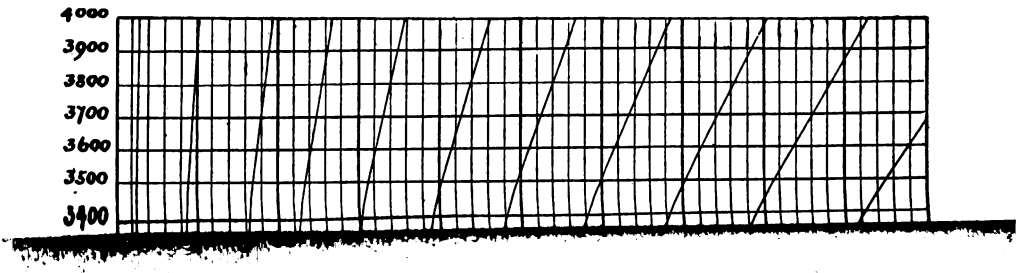
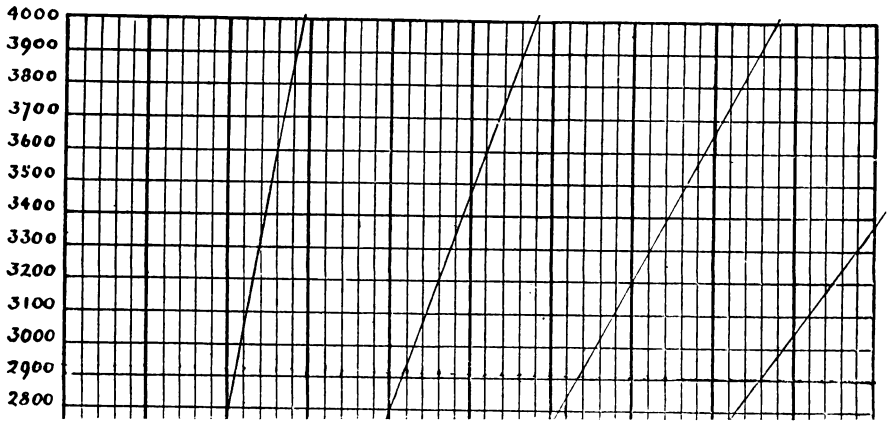


TABLE V.

COMMERCIAL EFFICIENCIES AT VARIOUS DISTANCES AND VARIOUS VOLTAGES OF MOTOR FOR MINIMUM TOTAL INITIAL COST OF TRANSMISSION PLANT.



tion of these two lines we find the radial line 65% which is the commercial efficiency of the system for minimum cost of plant From Table I we find in column 6 the efficiency nearest 65 is 64.8, to which corresponds a loss in the conductor of 20%. Hence 20% is the loss in the conductor for minimum cost of plant and $\frac{500}{.80}$ (.80 being the efficiency of the conductor from terminals of generator to terminals of motor) = 625 volts which must be the electromotive force of the generator. The loss in the wires equals 625 minus 500 = 125 volts or 20% of 625.

By the aid of the formulæ 3, 4, 5 and 6, the size, weight and cost of conductor can now be easily determined.

EXAMPLE 2.—Distance (including 5% for sag) 50,000 feet. We want to obtai n at least 55% (efficiency from generator pulley to motor shaft). What is the voltage to be employed at motor and generator for minimum cost of plant?

Solution.—We find the intersection of the 50,000 and 55% lines half way between the horizontal lines 1800 and 1900. The electromotive force of the motor must therefore be 1850 volts.

By looking at Table I we find that 32.5% loss in the conductor corresponds to 55% efficiency (54.68); hence

$$\frac{1850}{1-.325}=\frac{1850}{.675}=2741 \text{ volts,}$$

which is the electromotive force of the generator, and 2741 minus 1850 = 891 volts, which is the number of volts lost in the conductor, 891 volts being 32.5% of 2741.

EXAMPLE 3.—We wish to employ a motor of 1000 volts and get 60% efficiency of the electric system. At what dis tance can we place our factory from the power for minimum initial cost of transmission plant?

Solution.—From Table VI we find that the distance must be 20,000 feet (including 5% for sag) as the horizontal 1000 and the 60% efficiency lines intersect at the vertical 20,000 line.

These few examples we think will suffice to demonstrate the application of these tables.

As with fixed conditions of cost and efficiency of apparatus, the number of volts to get the minimum cost of plant is a function of the distance alone, and is independent of the electromotive force used at the motor we can calcu late a table somewhat like Tab'e VII. The values for

K, a, b, G and P are assumed as on page 26. Table VII really is nothing but an abstract of Tables V and VI.

TABLE VII.

Distance in feet plus 5% sag.	Volts lost in Conductor, for minimum cost of plant.	Distance in feet plus 5% sag.	Volts lost in Conductor, for minimum cost of plant.
1000	17.5	18000	315.
2000	35.	20000	350.
3000	52.5	25000	437.5
4000	70.	30000	525.
5000	87.5	35000	612.5
6000	105.	40000	700.
7000	122.5	45000	787.5
8000	140.	50000	875.
9000	157.5	60000	1050.
10000	175.	70000	1225.
12000	210.	80000	1400.
14000	245.	90000	1575.
16000	280.	100000	1750.

At the same time it seems somewhat startling that for the minimum cost of the installation under given conditions, as mentioned before, the volts lost in the conductor are dependent upon the distance alone.

It will be noticed that the number of volts lost at 1000 feet distance is 17.5, and that the other values are multiples of 17.5; for instance the number of volts lost for 10,000 feet is 17.5x10 = 175 volts, etc.

For practical reasons motors are not made for the voltage which may be theoretically the correct one, but certain standard voltages are adopted by the manufacturers. On pages 31. 32, 33, 34 and 35 we give tables for 220 volt, 500 volt, 800 volt 1000 volt and 1200 volt motors. These tables show at once the useful range of the various standard motors.

The number of volts lost in the conductor as given in Table VII added to the voltage of the motor, give the data in the second vertical column.

For instance. in Table VIII for the 220 volt motor for

1,000 feet, 220 volts+17.5=237.5.

2,000 feet, 220 volts+35.0=255.

3,000 feet, 220 volts+52.5=272.5.

The column "per cent. loss in conductor" and "per cent. efficiency of system" are calculated from these data.

For instance, 17.5 volts=7.4% of 237.5 volts, and the efficiency of the system is .81×.926=.75; where .81 stands for couple efficiency and .926 for 92.6% efficiency of the conductor, (100—7.4.)

TABLE VIII.

220 VOLT MOTOR.

DATA FOR MINIMUM COST OF INSTALLATION.

Distance in feet.	E. M. F. of generator.	% loss in conductor.	% efficiency of system.
1000	237.5	7.4	75.0
2000	255.	13.8	69.8
3000	272.5	19.4	65.3
4000	290.	24.2	61.4
5000	307.5	28.5	57.9
6000	325.	32.3	54.8
7000	342.5	35.8	52.0
8000	360.	38.9	49.5
9000	377.5	41.7	47.2
10000	395.	44.4	45.0
12000	430.	48.9	41.4
14000	465.	52.7	38.3
16000	500.	56.	35.6
18000	535.	58.8	33.3
20000	570.	61.5	31 2
25000	657.5	66.6	27.1
30000	745	70.8	23.8
35000	832.5	73.6	21.4
40000	920.	76.2	19.3
45000	1007.5	78.1	17.7
50000	1095.	80.	16.2
60000	1270.	82.7	14.0
70000	1445.	84.8	12.3
80000	1620.	86.4	11.0
90000	1795.	87.8	9.9
100000	1970.	88.9	9.0

Cross-section of conductor per mechanical horse power (delivered at motor shaft)= 4618.0 circular mils.

Weight of conductor per mechanical horse power and for each 1,000 feet of *distance*=28.86 lbs.

Weight of 1,000 feet of conductor per mechanical horse power=14.43 lbs.

TABLE IX.

500 VOLT MOTOR.

DATA FOR MINIMUM COST OF INSTALLATION.

Distance in feet.	E. M. F. of generator.	% loss in conductor.	% efficiency of system.
1000	517.5	3.38	78.3
2000	535.	6.54	75.7
3000	552.5	9.5	73.3
4000	570.	12.3	71.0
5000	587.5	14.9	68.9
6000	605.	17.4	66.9
7000	622.5	19.7	65.0
8000	640.	21.9	63.3
9000	657.5	24.	61.6
10000	675.	25.9	60.0
12000	710.	29.6	57.0
14000	745.	32.9	54.4
16000	780.	35.9	51.9
18000	815.	38.6	49.7
20000	850.	41.2	47.6
25000	937.5	46.7	43.2
30000	1025.	51.2	39.5
35000	1112.5	55.1	36.4
40000	1200.	58 3	33.8
45000	1287.5	61 2	31.4
50000	1375.	63.6	29.5
60000	1550.	67.8	26.1
70000	1725.	71.0	23.5
80000	1900.	73.7	21.3
90000	2075.	75.9	19 5
100000	2250.	77.8	18.0

Cross-section of conductor per mechanical horse power (delivered at motor shaft)=2032.0 circular mils.

Weight of conductor per mechanical horse power and for each 1.000 feet of *distance*=12.70 lbs.

Weight of 1,000 feet of conductor per mechanical horse power=6.35 lbs.

TABLE X.

800 VOLT MOTOR.

DATA FOR MINIMUM COST OF INSTALLATION.

Distance in feet.	E. M.F. of generator.	% loss in conductor.	% efficiency of system.
1000	817.5	2.2	79.2
2000	835.	4.2	77.6
3000	852.5	6.1	76.0
4000	870.	8.1	74.4
5000	887.5	9.9	73.0
6000	905.	11.6	71.6
7000	922.5	13.3	70.2
8000	940.	14.9	68.9
9000	957.5	16.5	67.6
10000	975.	18.0	66.4
12000	1010.	20.8	64.1
14000	1045.	23.4	62.0
16000	1080.	25.9	60.0
18000	1115.	28.3	58.1
20000	1150.	30.5	56.3
25000	1237.5	35.4	52.3
30000	1325.	39.6	48.9
35000	1412.5	43.4	45.8
40000	1500.	46.6	43.2
45000	1587.5	49.6	40.8
50000	1675.	52.1	38.8
60000	1850.	56.8	35.0
70000	2025.	60.5	32.0
80000	2200.	63.6	29.5
90000	2375.	66.4	27.2
100000	2550.	68.6	25.4

Cross-section of conductor per mechanical horse power (delivered at motor shaft)=1270.0 circular mils.

Weight of conductor per mechanical horse power and for each 1,000 feet of *distance* =7.937 lbs.

Weight of 1,000 feet of conductor per mechanical horse power=3 968 lbs.

TABLE XI.

1000 VOLT MOTOR.

DATA FOR MINIMUM COST OF INSTALLATION.

Distance in feet.	E. M. F. of generator.	% loss in conductor.	% efficiency of system.
1000	1017.5	1.72	79.6
2000	1035.	3.38	78 3
3000	1052.5	4.99	77.0
4000	1070.	6.54	75.7
5000	1087.5	8.1	74.4
6000	1105.	9.5	73.3
7000	1122.5	10.9	72.2
8000	1140.	12.3	71.0
9000	1157.5	13.6	70.0
10000	1175	14.9	68.9
12000	1210.	17.4	66.9
14000	1245.	19.7	65.0
16000	1280.	21.9	63.3
18000	1315.	24.0	61.6
20000	1350.	25.9	60.0
25000	1437.5	30 4	56.4
30000	1525.	34.4	53.1
35000	1612.5	38.0	50.2
40000	1700.	41.2	47.6
45000	1787.5	44 1	45.3
50000	1875.	46.7	43.2
60000	2050.	51.2	39.5
70000	2225.	55.1	36.4
80000	2400.	58.3	33.8
90000	2575.	61.2	31.4
100000	2750.	63.6	29.5

Cross-section of conductor per mechanical horse power (delivered at motor shaft)=1016.0 circular mils.

Weight of conductor per mechanical horse power and for each 1,000 feet of *distance*=6.349 lbs

Weight of 1,000 feet of conductor per mechanical horse power=3.175 lbs.

TABLE XII.

1200 VOLT MOTOR.

DATA FOR MINIMUM COST OF INSTALLATION.

Distance in feet.	E. M. F. of generator.	% loss in conductor.	% efficiency of system.
1000	1217.5	1.5	79.8
2000	1235.	2 8	78.7
3000	1252 5	4.2	77 6
4000	1270.	5.5	76.5
5000	1287.5	6.9	75.4
6000	1305.	8.1	74.4
7000	1322.5	9.2	73.5
8000	1340.	10 5	72.5
9000	1357.5	11.6	71.6
10000	1375.	12.6	70.7
12000	1410.	14 9	68.9
14000	1445.	17.0	67.2
16000	1480.	18.9	65.7
18000	1515.	20.8	64.1
20000	1550.	22.6	62.7
25000	1637.5	26.6	59.4
30000	1725.	30 5	56.3
35000	1812.	33.8	53.6
40000	1900.	36.9	51.1
45000	1987.5	39.7	48.8
50000	2075.	42.2	46.8
60000	2250.	46.6	43.2
70000	2425.	50 6	40 0
80000	2600.	53.8	37.4
90000	2775.	56 8	34.6
100000	2950.	59 3	32 9

Cross-section of conductor per mechanical horse power (delivered at motor shaft)=816.6 circular mils.

Weight of conductor per mechanical horse power and for each 1,000 feet of *distance*=5.291 lbs.

Weight of 1 000 feet of conductor per mechanical horse power=2.646 lbs.

The tables will now be readily understood, and only an explanation of the foot notes which will be found under each table is needed.

If we take formula 4:

$$M=\frac{16000\times N\times D}{E\times a\times V}$$

we find that all factors are known.

$\frac{D}{V}$ the relation of distance to volts drop for minimum cost of plant is a constant, viz:

$\frac{D}{V}=\frac{1000}{17.5}$ as V increases in direct ratio to D.

Again assuming "a" as .90, we get:

$$M=\frac{16000\times N\times 1000}{E\times .90\times 17\ 5}=\frac{1,016,000\times N.}{E} \quad \text{(Formula 9.)}$$

If we therefore divide the voltage of motor into 1,016,000 we get the number of circular mils for one horse power delivered at the motor shaft *for minimum cost of installation.*

If, for instance, 100 horse power is to be delivered by a 220 volt motor,

$$M=4618\times 100=461,800.$$

We also see that the cross-section of the wire for minimum cost of plant is the same for all distances, and depends only on the electromotive force of the motor and number of horse power to be delivered at the motor shaft.

We might, of course, state that the cross-section of the wire depends upon the *current* in ampe es instead of the electromotive force, but, as already explained, this is the same fact expressed in a different form.

In comparing Tables IX and XI (500 and 1000 volt motors) we see also that *with double the electromotive force or half the current, we require only half the cross-section of wire, and that we can deliver current twice the distance at the same percentage of loss in the wire.*

In the 500 volt table, for instance, we find at the distance of 10,000 feet 25.9 per cent as the loss in the conductor and in the 1000 volt table we find at a distance of 20,000 feet 25.9 per cent as the loss in the conductor.

If we now consider formula 5,

$$\text{Weight in lbs.} =\frac{N\times D^2}{10\times E\times a\times V}$$

we can again consider $\frac{D}{V}=\frac{1000}{17.5}$ as a constant,

and assuming a = .90 we get

$$Wt=\frac{1\times1000\times N\times D}{10\times.90\times17.5\times E}$$

$$Wt=\frac{6.3492\times D\times N}{E} \qquad \text{(Formula 10.)}$$

If we now assume D=1000 and N=1 we get:

$$\text{Wt. in lbs.}=\frac{6349.2}{E}$$

which is the weight of the conductor per horse power (at motor shaft) for each 1000 feet of distance for minimum cost of plant.

EXAMPLE: We have to transmit 100 horse power 5000 feet, and use a 220 volt motor.

Solution: Table VIII gives weight of conductor per horse power per 1000 feet distance=28.86 pounds. Hence weight of conductor=28.86×100×5=14,430 pounds.

It will be noticed that the weight first given under each table is for each 1,000 feet of distance, including outgoing and return wires.

The weight per 1,000 feet of conductor per horse power of course is one-half of that for 1000 feet distance.

The data relate to the weight of *bare copper* wire.

Quite a number of other interesting data may be deduced from the foregoing tables.

If we compare the 500 volt, 800 volt and 1000 volt tables (IX, X, XI) we find, for instance, the weight of conductor per horse power for:

500 volt, 10,000 ft. dist. 25.9% drop=10×12.7=127 lbs.
800 " 16,000 " " 25.9% " =16×7.937= 27 "
1000 " 20,000 " " 25.9% " =20×6.349=127 "

In other words, we see that *for minimum cost of plant the total weight of the conductor per horse power delivered by the motor shaft remains the same at a certain percentage of loss in the conductor regardless of the voltage of motor and the distance.*

As a certain efficiency of the electric system corresponds to a certain drop in the conductor (see table I) the total weight of the conductor will also be the same at a certain efficiency of the electric system, regardless of voltage and distance.

This law really follows from some of the foregoing considerations, viz., with double the electromotive force we require only half the cross-section of wire and we

can reach twice the distance at the same percentage of loss in the wire.

If the weight of the wire at 220 volts and 3000 feet distance, for instance, is 86 pounds, at 440 volts and 6000 feet distance. wire of half the cross section will weigh, of course, $\frac{86\times 2}{2}=86$ pounds as before.

These considerations allow us to construct a table somewhat like table XIII.

After determining the best commercial efficiency of the electric transmission system for least cost of plant from Tables V or VI we can readily answer the question as to the weight or cost of the copper conductor.

If, for instance, we decided on a commercial efficiency of 60 per cent. we find by looking at Table XIII the intersection of the curve and the vertical 60 per cent. line, a little above the horizontal 125 pounds line. The weight of copper conductor per horse power (delivered at motor shaft). therefore is about 127 pounds, and the cost (at 25 cents a pound delivered at the poles) about $32.

The commercial efficiency of the electric transmission system considering Table I, is evidently:

$$l=a\times b\times x$$

What is the commercial efficiency of the whole system:

a is commercial efficiency of motor,

b " " " " generator,

and x " " " " conductor.

The efficiency of the conductor, of course, equals 1 minus the percentage of loss (written as a decimal fraction.)

Assuming a=.90 and b=.90 we get:

$$l=.81\times(1-\%)=.81-.81\times\%$$

where % stands for per cent. loss in the conductor (written as decimal fraction). Hence:

$$\%=\frac{.81-l}{.81}$$

or in words, the per cent lost in the conductor equals the couple-efficiency minus the commercial efficiency of the system divided by the couple-efficiency.

This shows that assuming a certain couple-efficiency, the per cent. lost in the conductor is a function of the commercial efficiency of the system only.

We can further deduce from Table I that

$$l=\frac{E}{E+V}\times a\times b$$

or in words, the commercial efficiency of the system equals the electromotive force of the motor divided by the electromotive force of the generator multiplied by the couple-efficiency.

Assuming again $a \times b = .81$

we get: $l = \frac{E}{E+V} \times .81$

From this formula we deduce:

$$E = \frac{l \times V}{(.81 - l)}$$

If we now insert this value into formula 10, viz:

$$Wt. = \frac{6.3492 \times N \times D}{E}$$

we get:

$$Wt. = 6.3492 \times N \times \frac{D\,(.81 - l)}{V \times l}.$$

If we now insert the value for $\frac{D}{V} = \frac{1000}{17.5}$

into this formula we get:

$$Wt. = \frac{6.3492 \times 1000 \times (.81 - l) \times N}{17.5 \times l}$$

$$\text{Wt. in lbs.} = 362.81 \times \frac{(.81 - l)}{l} \times N. \quad \text{(Formula 11).}$$

If we now substitute the value:

$$l = .81 \times (1 - \%)$$

in formula 11 we get:

$$\text{Wt. in lbs.} = 362.81 \times \frac{\%}{1 - \%} \times N. \quad \text{(Formula 12).}$$

This formula shows conclusively that for minimum cost of plant the weight of the conductor depends only on the percentage of loss in the conductor and the number of mechanical horse power delivered by the motor.

Capacity and Cost of Generators for Minimum Cost of Plant.

We demonstrated in a previous chapter that if we want to deliver N mechanical horse power at the motor shaft, $\frac{N}{a}$ electrical horse power must be delivered to the motor terminals, where "a" is the commercial efficiency of the motor.

The generator, of course, must generate $\frac{N}{a}$ electrical horse power plus horse power lost in the conductor.

The efficiency of the conductor, however, equals $1-\%$ ($\%$ written as a decimal fraction ; hence

Capacity of generator $= \frac{N}{a \times (1-\%)}$ electrical horse power. (Formula 13.)

EXAMPLE: Assuming a=.90, N=1, and loss in the conductor=.10 (10%) we get:

Capacity of generator $= \frac{1}{.90\,(1-.10)} = 1.2345$ electrical horse power.

As a matter of fact column 4 of Table I gives at once these ratios for various percentages of loss in the conductors.

We may again substitute "l" (commercial efficiency of electric system) in lieu of "$\%$" (loss in the conductor).

It is evident that the number of mechanical horse power required to drive the generator equals the number of mechanical horse power delivered at the motor shaft divided by the efficienoy of the system or $\frac{N}{l}$. We want to know, however the capacity of the generator in electrical horse power, and must therefore multiply this value with the efficiency of the generator. Hence:

Capacity of generator $= \frac{N \times b}{l}$ electrical horse power, (Formula 14.)

where l stands again for commercial efficiency of the system and b for the commercial efficiency of the generator.

EXAMPLE: If we assume b=.90, l= 729 and N=1 we get: Capacity of generator $= \frac{1 \times .90}{.729} = 1.2345$ electrical horse power as before; this value coincides again with the data in Table I.

If we now assume the cost in dollars of generator per electrical horse power, delivered at generator terminals, as 'G" (which includes cost of freight, haulage, foundation, electrical station instruments and labor in construction) we get from formula 14:

Cost of generator in $ for mechanical H.P. delivered by the motor shaft $= \frac{N \times b \times G}{l}$ (Formula 15.)

Assuming G=$45 and b=.90 we get for minimum cost of plant:

Cost of generator in $= $\frac{N \times 40.5}{l}$

We can now easily understand Table XIV, which was calculated on the basis of this formula

We can also easily determine the approximate *minimum* cost of an electric power transmission plant.

EXAMPLE: We have decided on a motor of, say, 600 volts; the distance is 14,000 feet; 100 horse power is to be delivered by the motor.

Solution: We find from Table VI the best efficiency of the system to be .575 (57.5%). Table XIII tells us at once: Cost of copper per horse power=$37.00, and Table XIV: Cost of generator, per horse power, $70.50.

If we now assume the cost of the motor at $50.00 per mechanical horse power, delivered (including freight, electrical instruments, and construction), we get cost per horse power=$37.00+$70.50+$50.00=$157.50; or for 100 horse power=$15.750 00, as cost of electrical installation, including dynamos, motors and copper wire. To this, we must add the cost of power plant, and cost of poles and line construction, which can be easily determined according to local conditions.

In determining the cost of power plant (steam or water) per mechanical horse power delivered by the motor, it must not be forgotten to determine the number of mechanical horse power to be delivered at the generator pulley.

This power, of course, equals mechanical horse power delivered by motor, divided by the efficiency of the electric system.

In the present case: Mechanical horse power to be delivered at generator pulley $= \frac{100}{.575} = 173.9$ horse power, or in round figures 174 horse power.

We can verify this by consulting Table I. We find in the sixth column: 58.7% and 56.7%; corresponding to these efficiencies we find in the fifth column: 1.70 and 1.76 mechanical horse power, to be delivered at the generator pulley.

The mechanical horse power at generator pulley corresponding to 57.5% efficiency, is therefore, about 1.74, or for 100 mechanical horse power delivered by motor, 174 mechanical horse power, is required at the generator pulley.

If we call P the cost in dollars of (steam or water) power plant, delivered at generator pulleys, then:

Cost in dollars of power plant (per mechanical horse power delivered by motor) $= \frac{P \times N}{l}$

Assuming, for instance, P=$25.00 (water power) we get in this example:

Cost of power plant $= \frac{P \times 100}{.575} =$ $4,350.00 which shows that the cost of the power plant for *one* mechanical horse power at the motor shaft is $43 50.

If we further assume the cost of po'es and line erection at, say, $60.00 per 1000 feet distance, we get as cost of line construction $60 \times 14 =$ $840.00.

The total cost of plant would therefore be:

Cost of bare copper conductor, - -	$3700.00
" " generators, - - - - -	7050.00
" " motors, - - - - - -	5000.00
" " power plant (water). - - -	4350.00
" " poles and construction of pole line,	840.00
Total, - - -	$20,940.00

This is the minimum cost of a transmission plant to deliver 100 mechanical horse power at the shaft of a 600 volt motor, at a distance of 14,000 feet.

This one example, we think will suffice to enable our readers to determine the approximate minimum cost of any electric transmission plant, with the aid of the preceding tables.

It might also be very convenient to construct a table somewhat like Table XV, which will give the reader a recapitulation of the formulæ, for the minimum initial cost of plant.

We may express the principles governing the minimum cost of a transmission plant, in the following rule:

For minimum initial cost of plant, and assuming certain prices per horse power of motors, generators and power plant (all erected and ready for operation) and assuming a certain price per pound of copper, (delivered at the poles), the total cost of the plant, excluding line construction, is a constant for a certain efficiency of the electric system, no matter what the electromotive force of the motor and the distance may be.

At a given efficiency of the electric system the electromotive force of the motor and distance will increase and decrease in the same ratio.

TABLE XIII.

WEIGHT OF BARE COPPER CONDUCTOR PER MECH. H. P. DELIVERED BY MOTOR SHAFT FOR MINIMUM COST OF PLANT.

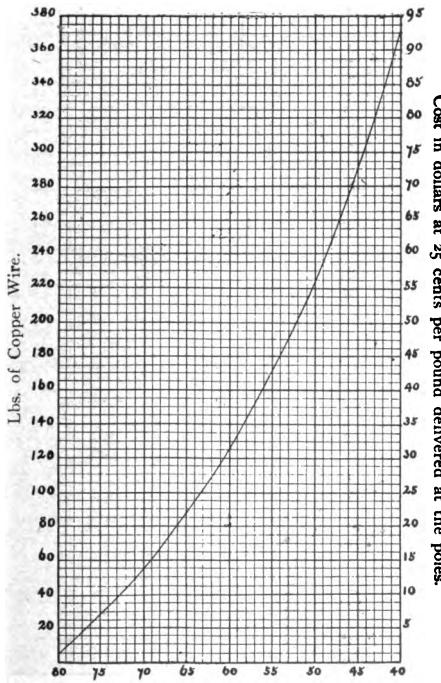
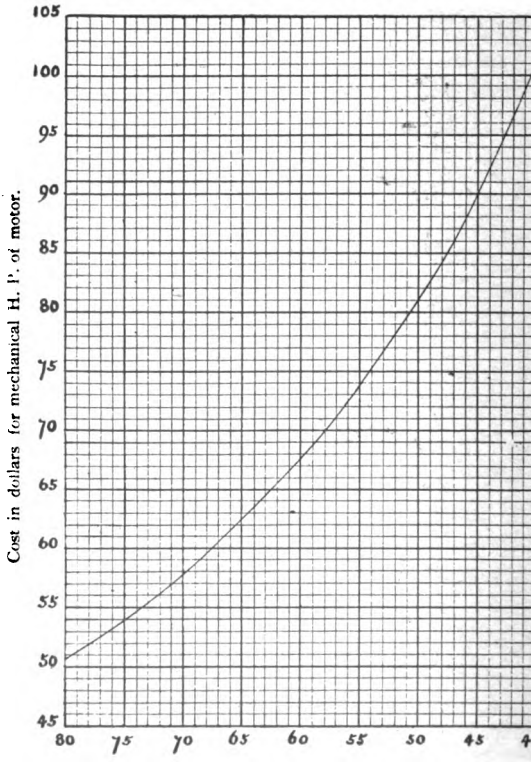


TABLE XIV.

COST IN DOLLARS OF GENERATOR PER MECH. H. P. DELIVERED BY MOTOR SHAFT.



Cost of generator per electrical H. P. delivered at generator terminals is assumed as $45.00. (This) includes cost of freight, haulage, foundations, electrical station instruments, and labor in construction.

ELECTRIC POWER TRANSMISSION DATA FOR MINIMUM INITIAL COST OF PLANT

TABLE XV.

(Per Mechanical H. P. delivered by Motor.)

BASIS OF CALCULATIONS.				MOTOR (90% Efficiency)	CONDUCTOR (Bare Copper.)				GENERATOR (90% Efficiency.)			COST IN DOLLARS of Electric Plant Exclusive of Line Erection.					REMARKS.
D	E	I		$\frac{1}{.90}$	$\frac{.746}{E}$	V	%	C M	WT. lbs.	$\frac{.90}{I}$	$\frac{1}{I}$	E+V		K × .693 × $\frac{.693}{I}$ lbs.		$\frac{P}{I}$	
Distance in feet	E. M. F. at Motor Terminals	Efficiency of Electric System.		El. H. P at Terminals	Amperes.	Volts lost in Conductor.	% lost in Conductor	Cross-Section in C M.	Weight in lbs.	Output in El. H. P.	Mech. H. P. required to drive.	E. M F. at terminals.	Motor.	Conductor.	Generator.	Total of Electric Plant.	Power Plant (Water.)	GRAND TOTAL. (Excluding Line Erection.)	
5,000	500	68 9		1.11	1.492	87.5	14 9	20'92	63. 5	1. 29	1. 45	587. 5	50	15. 87	58. 54	104. 41	36. 28	160. 69	
10,000	500	60.0		1.11	1.492	175.0	25 9	2092	127. 0	1. 50	1. 67	675	50	31. 75	67. 50	149. 25	41. 67	190. 92	
16,000	500	51.9		1.11	1.492	280. 0	35. 9	2092	203. 2	1. 74	1.93	780	50	50. 80	78. 04	178. 84	48. 17	227. 01	
5,000	1,000	74. 4		1 11	0.746	87.5	8.1	1016	31. 75	1 21	1 34	1087 5	50	7 94	54. 43	112. 37	33. 60	145. 97	
10,000	1,000	68.9		1 11	0.746	175.0	14. 9	1016	63. 5	1. 31	1. 46	1175	50	15. 88	58. 78	124. 66	36. 28	160. 94	
16,000	1,000	63 3		1 11	0.746	280. 0	21. 9	1016	101. 6	1. 42	1. 58	1280	50	25. 40	68. 98	139. 38	39. 43	178. 87	
25,000	1,000	56. 4		1 11	0.746	437. 5	30. 4	1016	158. 7	1 59	1. 77	1437. 5	50	39. 69	71. 81	161. 49	44. 33	205. 8	
35,000	1,000	50..		1 11	0.746	612. 5	38 0	1016	222. 2	1. 79	1 99	1612. 5	50	55. 54	80. 65	185. 92	49. 80	235. 72	
35,000	4,000	70.2		1 11	0 1863	612.5	13 3	254	55. 6	1 28	1. 42	4612 5	50	13. 90	57. 69	121. 59	36. 61	167. 20	
70,000	4,000	62		1.11	0.1865	1125	23. 4	254	111. 1	1. 45	1. 61	5225	50	27 78	65 32	143. 10	40. 32	183. 42	
100,000	4,000	56 4		1.11	0.1865	1750	30. 4	254	158. 7	1. 59	1. 77	6750	50	39 69	71. 81	161. 49	44. 33	205. 82	

G=Cost of Generator delivered and erected, incl. Electrical Instruments per El. H. P. delivered at Generator terminals=$45.00.

P=Cost of Power Plant (Water) erected, per mech. H. P. delivered at Generator Pulleys=$25.00.

K=Cost in Cents of BARE Copper Wire per lb., delivered at the Poles=25 cents.

We see, for instance, in Table XV, that the cost of plant per horse power delivered by motor at 1000 volts and 25,000 feet distance and at an efficiency of 56.4% is $205.82.

We find that the cost is the same at 4000 volts, 100,000 feet distance, and the same efficiency of 56.4%.

While the cost and efficiency in both cases are the same, with an electromotive force four times greater we can reach four times the distance.

CHAPTER VII.

Alternating Current Transmission System.

So far we have considered only electric power transmission by means of *continuous* currents.

Of course, it is also possible to employ the ordinary alternating transformer system as used for the distribution of incandescent light, for the transmission of power.

The high tension alternating current may be carried from the power station to points of distribution and there converted by means of transformers to a lower potential.

The use of the alternating current, however, would necessitate the introduction of alternating motors, which are not yet extensively used in practice on account of their imperfection. The London *Electrical Review* remarked in an editorial referring to Kapp's Cantor Lectures:

"For several years past, from the days of Prof. Ferrari's investigations, which were followed by those of Tesla, Zipernowsky, and a host of imitators, we have periodically heard of the question of alternating current motors being solved. More than once have we questioned the solution of this difficult problem and certain enthusiastic inventors had the free use of our pages for the ventilation of their pet theories, yet we are not aware of the existence of any commercially successful motor, one which gives a good efficiency with a reasonable speed at variable loads, and which is entirely self-starting. If those gentlemen, who then contradicted our assertions, can now show us any radical improvements, we shall be glad to record them. Mr. Kapp spoke of several types of *small* alternating current motors with two and three wires, but he has not pointed out any successful applications."

In view of these remarks we may give a description of Brown's experiments. He undoubtedly succeeded in transmitting electrical energy by alternating currents of from 30,000 to 40,000 volts, but the question arises what can he do with this energy at the point of distribution? He can use it only for incandescent lighting on a large scale, and for power purposes on a very small scale.

As long as alternating motors cannot be made self-starting and automatic in regulation, of high commercial efficiency, and be built in as large units as continuous current motors, Mr. Brown's experiments in long distance power transmission will remain simply "experiments."

Fig. 2 is a diagram representing the experimental plant in in which *a* is an alternating current dynamo with a difference of potential of 130 volts, *b* is a fuse block for the protection of the dynamo, *c* is a double pole switch with fuse attachments, *d* and *e* represent respectively an ammeter and a voltmeter, *f* is a main converter, receiving in its primary coil the entire current from the dynamo. The winding of the secondary coil of this converter is 300 times that of the primary, producing, with due allowance for loss of energy in transforming, a current of over 30,000 volts. The measurement of the electromotive force of this current is made by a Thomson electrostatic voltmeter *g*. The current from the converter is carried overhead on a pole line. The conductors run out and back again on the same poles, a distance of 2½ miles, making the total length of the line five miles. The wires terminate in another converter *i*, by which the electromotive force is again reduced. The current in the tertiary wire can be brought down as low as 50 volts and is measured by the ammeter and voltmeter *k* and *l*; o_1, o_2, o_3, are three banks of lamps of 50, 65 and 100 volts, controlled by switches n_1, n_2, n_3.

To determine the influence of high tension current on telephone service, a double telephone line was extended between the outgoing and the incoming wires on the same cross arms.

Mr. Brown obtains the high insulation on line and in transformers by the application of oil, which is known to be one of the best non-conductors of electricity. It has the property of filling all pores of insulating materials, such as cotton linen, paper, etc., and excluding air and moisture. Oil is therefore capable of insulating against the influence of the atmosphere.

A well known experiment best demonstrates this fact.

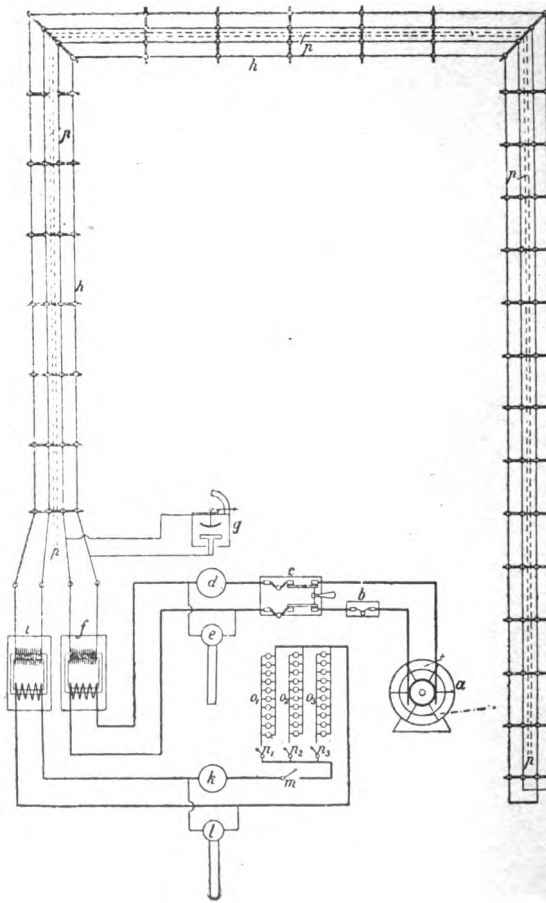


FIG. 2.—ALTERNATING TRANSMISSION PLANT.

We twist the ends of two cotton braided wires together, and put them in a glass filled with thick resinous oil (see Fig. 3), which then is heated for a few hours in order to expel all air and water out of the cotton and the oil. Over the wires are slipped two glass tubes which extend above the surface of the oil. We may now pour a layer of water on the oil. If we then connect the upper ends of the wire to the poles of a Rhumkorff induction coil or á frictional

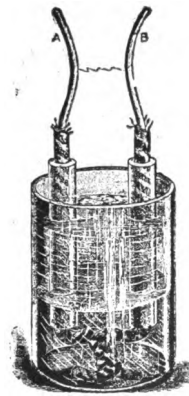


FIG. 3.—OIL AS AN INSULATOR.

(influence) machine, we can obtain sparks between these wires a few inches in length without puncturing the cotton insulation submerged in the oil.

This is without doubt proof of the highly insulating properties of certain oils.

The first application of oil for this purpose was made probably by Brooks in his underground conduits. He drew common cotton insulated wires through pipes and then filled these with oil.

Mr. Brown of Oerlikon proposed the application of oil as insulating material for transformers for very high potentials.* In his experiments the whole transformer was put

* Prof. Elihu Thomson secured a U. S. patent on the same process. His patent application was filed May 9, 1887. (*Western Electrician*, April 18, 1891.)

in a cast iron box, which was then filled with oil. This was kept heated for hours to a temperature of 150 degrees centigrade. Brown claims that through this process he obtains the following advantages:

The oil fills and penetrates all pores and fissures of the apparatus, entirely displacing air and moisture and covering the whole transformer. The possibility of puncturing the insulation is, of course, greatly diminished, and the transformer protected against influences from exterior sources, such as atmosphere, dust, etc.

The apparatus exposes nothing but a surface of oil, and the fact that the oil remains a compact mass, makes it very much more fit for this purpose, than for instance, paraffine, which will crack and absorb moisture through the fissures.

Of course, it is necessary in the first place to construct and insulate the transformer in the very best manner possible, and not depend upon the oil insulation solely.

CHAPTER VIII.

Line Construction.

For potentials say not over 500 volts, the line, whether overhead or underground, must be constructed in the same manner as for the distribution of electric current from central lighting stations.

For long distance transmission of electrical energy with high potentials the *overhead* system is undoubtedly the most practical and economical.

Sir W. Thomson recently pointed out that there can be not the slightest objection raised to the adoption of *bare* wire carrying high potentials outside of densely populated districts.

Overhead lines can be constructed in such a manner with *bare* wires carrying very high voltages that practically perfect insulation may be obtained.

The points of support, of course, must be highly insulated in order to prevent leakage.

The glass or porcelain insulators which are now in common use are very good insulators only when they are perfectly clean and when the atmosphere is dry. In moist atmosphere, the insulator will be covered with a film of moisture outside and inside, and the current will pass through the film to earth and cause what is called "surface leakage."

This consideration led Johnson & Phillips of London, to construct their fluid insulators. These insulators are made of glass, porcelain or brownware, and are provided on the inside with a trough containing oil. (See Fig. 4.)

The oil used for this purpose is thin and of a low specific gravity in order to allow the water to sink to the bottom of the receptacle.

If any condensation of moisture takes place now, it does not form a film over the whole surface of the oil, but it forms little drops of moisture which have no connection with each other, and which will sink to the bottom of the trough just as soon as they have grown to a certain size.

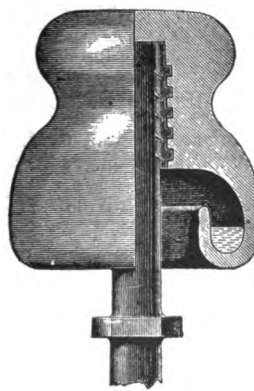


FIG. 4.—OIL INSULATOR.

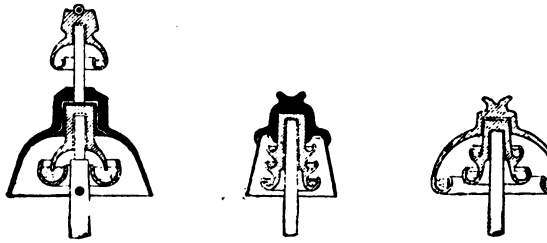
We must not imagine that oil will be displaced by the water in a very short time, because the amount of moisture which is condensed under the bell of an insulator is very little during a year, and it will be at least two or three years before the insulators will need refilling with oil.

The insulating properties of these fluid insulators are extremely high.

Prof H. F. Weber of Zurich made some experiments in 1887 on the conductors put up for the power transmission plant, Kriegstetten-Solothurn; the conductors had a total length of 24 kilometers and were fastened to fluid insulators.

Prof. Weber could not detect the least leakage, even with a current of 2,000 volts passing through the wire. The insulation of the line was practically perfect.

For the conveyance of currents of say 20,000 or 30,000 volts however, even these insulators would not be quite



FIGS. 5, 6 AND 7.—OIL INSULATORS FOR HIGH POTENTIALS

sufficient. Combinations, however, as shown in Figs. 5 to 7, would increase the insulation resistance to any desired degree.

These insulators could be easily protected against malicious throwing of stones or other destruction, by providing each with a bell shaped iron cover.

The objection might be made that such insulators would need constant attention, as they must be cleaned and refilled

FIG. 8.—SYPHON FOR FILLING OIL INSULATORS.

with oil at certain intervals. This, however, does not seem a great disadvantage, for the reason that the fastening of wires carrying such high potentials should be inspected from time to time.

At any rate, these considerations will show that one of

the objections made to transmission of high potential currents, viz., the enormous leakage, does not hold good, as even with the highest potentials good line construction will prevent practically any leakage.

Fig. 8 shows a siphon for filling oil insulators.

Lightning arresters must be applied, not only to each end of the line, but at certain distances between the power and motor station. These lightning arresters must be automatic so that they will rupture an arc in case it should be set up. Such an automatic lightning arrester can, for instance, be constructed by having the lightning pass over two carbon plates a small distance apart, and are automatically separated from each other by the dynamo current rupturing the arc; then, of course, the plates should *automatically* return to their former position.

There are, however, other lightning arresters constructed on different principles, which automatically prevent the formation of an arc. (Prof. E. Thomson, Wirt, Westinghouse, etc.)

CHAPTER IX.

Classification of Motors.

We have already mentioned that there are three principal systems of motor service, viz.: The constant current, the constant potential (pressure) systems, and a system in which both current and pressure vary.

Motors may be classified as

1. Series motors in which the field coils are connected in series with the armature. (See Fig. 9.)

2. Shunt motors in which the field coils are connected in shunt or parallel with the armature. (See Fig. 10.)

Compound-wound motors which have both the shunt and the series winding.

This latter class may be again subdivided into:

3. Differential motors in which the series coil magnetizes the field magnets in opposite polarity to the shunt coil. (See Fig. 11.)

4. Cumulative motors in which the series coil magnetizes the field magnets in the same polarity as the shunt coil. (See Fig. 12.)

Generators are electric machines in which mechanical energy is converted into electrical energy, and motors are electric machines in which electrical energy is re-converted

into mechanical energy. As a matter of fact any dynamo wound and connected for working as a generator of direct currents may be used as a motor.

1. A *Series Dynamo* running right-handedly will, when

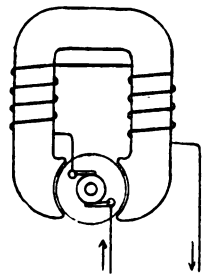


FIG. 9.—SERIES MOTOR.

supplied with a current from another source, run left-handedly against the brushes.

To adjust a series dynamo for motor service requires the reversing of the connections of the armature or the field

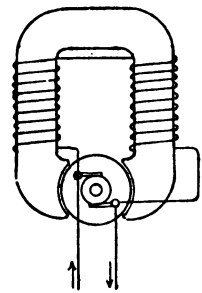


FIG. 10.—SHUNT MOTOR.

coils, or the brushes may be reversed and given a lead in the opposite direction of rotation.

2. A *Shunt Dynamo*, when operated as a motor, will run in the same direction as when operated as a generator.

If the current passes through the armature in the same direction as before, it flows in the opposite direction through the field, and *vice versa*.

3. A *Compound-Wound Dynamo* will run as a motor

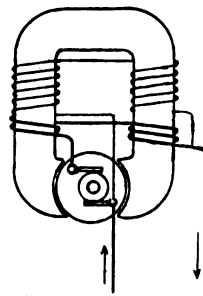


FIG. 11.—DIFFERENTIAL MOTOR.

"with the brushes" if the shunt coil is more powerful than the series coil and "against the brushes," if the series coil is more powerful than the shunt. If the *generator* is so connected that the series coil is *cumulative*, i. e., mag-

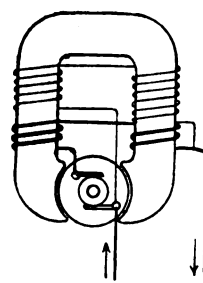


FIG. 12.—CUMULATIVE MOTOR.

netizes the field magnets with the same polarity as the shunt coil, it will act the same as a *Differential Motor* as the series coil will now magnetize the field magnets in the opposite direction,

If it is desired to run the *compound generator* as a *cumulative motor*, the connections of the series coil must be reversed.

All these facts will be easily understood by studying the diagrams of series, shunt and compound dynamos, which will be found in almost any elementary handbook on electri lighting.*

Of course, any of the four different motors may be used on any of the three different systems of motor service, which would allow of twelve principal combinations. We shall consider, however, only those combinations which are in general practical operation.

a. The Series Motor on the Constant Current (Arc Light) System.

The large number of arc light circuits which are operated on a constant current system naturally caused a demand for

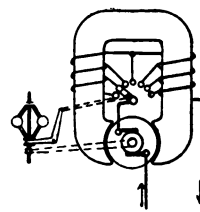


FIG. 13.—AUTOMATIC ARC CIRCUIT MOTOR.

motors which could be placed in the circuit like arc lamps.

There are, however, numerous drawbacks. A series motor placed on an arc circuit can deliver power in *small* units only.

The number of amperes of such a circuit is constant and small, and consequently the electromotive force at the terminals of the motor increases with the number of horse power.

Let us consider a 10 ampere circuit and a motor requiring 20 electrical horse power.

One electrical horse power requires $\frac{746}{10}$=74.6 volts as there are 746 volt-amperes (watts) to one electrical horse power. Hence 20 electrical horse power will require 74.6×20 = 1,492 volts pressure at the terminals of the motor.

*See "Dynamo Tenders' Hand-book."

Such a voltage, however, must be considered very high in a comparatively small motor running in a manufacturing establishment. If 20 electrical horse power require 1,500 volts, of course, 40 electrical horse power would require a generator of over 3,000 volts electromotive force, which we may consider as the limit. We see, therefore, that arc light circuits are adapted to run motors only on a small scale and in smaller units. To still better illustrate this fact we reproduce here data taken from the catalogue of a prominent company.

SELF-REGULATING MOTORS FOR ARC LIGHT OR OTHER CONSTANT CURRENT CIRCUITS.

Horse Power.	Consumption in Volts with Current Strength of						Weight in pounds.	Revolutions per minute.
	7 Amperes.		10 Amperes.		20 Amperes.			
½	60	volts.	43	volts.	22	volts.	107	2500
1	120	"	85	"	43	"	157	2100
2	240	"	170	"	85	"	305	2000
3	350	"	244	"	122	"	330	1900
5	610	"	425	"	213	"	500	1800
7½	900	"	625	"	313	"	750	1700
10	1210	"	850	"	425	"	1008	1600
15	1815	"	1275	"	638	"	1500	1500

Another disadvantage of the series motor on a constant current system is the fact that the motor needs special apparatus for the regulation of its speed.

Without such a regulator the motor will be liable to "race" and "run away" when the load is decreased.

Automatic regulation may be effected in many ways. The two principal modes are the automatic shifting of the brushes and the cutting in or cutting out of parts of the series coils of the field.

Fig. 13 represents an arc light motor of the latter type. A centrifugal governor operates a sliding contact. If the motor commences to speed up with decreasing load one or more of the field coils are cut out, thus reducing the strength of the field magnets and consequently the speed of the armature; if the motor is decreasing in speed the opposite action will take place, in each case keeping the speed of the armature normal.

b. The Series Motor on the Constant Potential System.

The series motor on the constant potential system is capable of exerting a great rotary effort and doing a large

amount of work at a slow speed. The range of speed for different loads is great, and the motor is unfitted for work where a uniform speed is required.

For railway and similar work, however, this motor is universally used as it admits of easy *hand* control and the strength of its field may be made the same for slow and high speeds, small and large currents.

Besides, the rotary motion of the armature of a series motor is easily reversed, either by reversing the brushes and giving them a lead in the opposite direction, or by reversing the connections of either the armature *or* the field magnets.

The motor is used generally where a certain amount of work must be done with varying speeds under *hand* control.

Hence it is advantageously used for railways, cranes, turntables, elevators, hoists etc.

c. The Series Motor on a System in which both Current and Pressure are Variable.

It would seem at first sight that the series motor whose speed is difficult to govern on either the constant current or constant pressure system, would behave worse on a system in which both current and pressure are variable. Such, however, is not the case.

Frank J. Sprague pertinently remarks as follows :

"There are some special features in long distance transmission meriting notice and one of these is the appl cation of series dynamos and motors where permissible.

"While this type of motor does not ordinarily play a part in central station distribution, it has a field in the special transmission of power from one point to another in single units or batteries, and the very characteristics which in the machine taken singly destroy its regulation, become the means for determining a very perfect regulation when coupled together. Suppose two such machines are running, one as a dynamo and the other as a motor, let us see just what happens when the load varies, and in this way perhaps get a clearer idea of the meth ds and conditions of self-regulation than by a mathematical investigation. We will assume that a certain amount of power is being transmitted, and the generator is being driven at a constant speed, its electromotive force being, say, E, that of the motor, e, and the resistance of the circuit R. The current then flowing is $\frac{E-e}{R}$, which passing through the field and

armature of the motor produce the requisite torque. Suppose the load on the motor is increased, it tends to slow down. Its counter electromotive force will consequently be decreased, and even with the same initial electromotive force the current would be increased, thus increasing the torque and preventing any further reduction of speed.

"This increase of current, however, acts upon the dynamo, increasing its electromotive force, which tends to further augment the current, giving the motor more than sufficient torque to take care of its load. The only thing left for the motor to do is to increase its speed, and by reducing the current restore the equilibrium, and if the machines are properly proportioned this recovery of speed will be exact. It is evident that the current being the same in both machines, the ratio $\frac{e}{E}$ representing the electrical efficiency, likewise represents the relative watt capacity of the two machines. It is further evident that this ratio is constant, and in order that this may be so, since both machines are excited by the same current and are run at the same speed, the characteristics of the two machines must be similar between the limits of automatic regulation. We see also that the initial electromotive force, that of the motor and the current, all vary, not by like increments, but in the same ratio. This same method of regulation can be applied to a battery of machines in series transmitting power to another battery of machines, likewise in series. In this method of transmission, where large powers, long distances, and necessarily high potentials are used, it is advisable to divide the generators and motors into a battery of machines of identically the same weight and character, the number of the machines being the ratio of the electrical efficiencies, the generators all to be driven from the same line of shafting, and the motors to drive on a common line and the current to pass through all the machines in series. This I consider one of the best methods when dealing with large powers and high potentials and single units of generation and recovery, especially where automatic regulation is required.

"Its advantages are manifest. One of the greatest difficulties which we have in dynamo-electric construction in closed circuit machines, especially where using the drum system of winding, is that of securing perfect insulation when high potentials are used.

"A potential of 1,000 or 1,200 volts seems as high as it is now advisable to go in machines of this type, when cur-

rents of any maguitude are to be used. In the transmission of electricity reliability is an essential and a potential of 3,000 or 4,000 volts, distributed over three or four machines in series, is, despite the increased number of machines, far less liable to cause failure than where put into one machine of the aggregate size of the four. In the event of the breaking down of one machine, the units may be so proportioned that, by a corresponding change in the units at the other station or a proper variation of the regulating shunt to the fields, it becomes quite possible to continue automatic operation. As an illustration of this distribution of machines if wishing to use dynamos and motors of an electrical efficiency of about 95 per cent., a commercial efficiency, each of about 90 per cent., and with about 60 per cent. as the commercial efficiency of the circuit, I would, with a distance of about nine miles, use five series machines identical in construction, each wound for about 1,200 volts, three of which machines would be driven by a common line of shafting at the generating station and two driving on a common line of shafting at the receiving station "

So much for Sprague. On the other hand we read in the London *Electrical Review* relating to Kapp's Cantor Lectures the following:

"In the case of series motors driven by series dynamos, automatic regulation can only be obtained within certain limits as was shown by the aid of characteristic curves, at the lower portion of which 'racing' of the machines would take place after a sudden and considerable reduction of the load."

This does not seem to quite coincide with a review in the same paper on Kapp's third edition of "Electric Transmission of Energy," in which the following paragraph occurs:

"The author describes a novel transmission plant which he has seen tested in the works of the Allegemeine Elektricitaets Gesellschaft at Berlin. It is a self regulating device which allows of extreme variations of load giving constant motor speed; the only condition to be observed is, that the generator is driven at constant speed. It follows a theory, explained in the book according to which a certain relation between the characteristics of the two machines *insures constant speed* of the motor *at all loads*. The plant described consists of a 20 kilowatt generator of the Edison type, working with a normal current of about 25 amperes, a

motor of the same type and leads connecting the two, having a resistance of 1.25 ohms. The generator runs at 1,000 and the motor at 800 revolutions per minute. The general dimensions of motor and generator differ comparatively little. The magnet cores of the motor are slightly smaller in diameter than those of the generator, and the yoke is somewhat lighter. The armature cores are alike in both machine, viz.: 11⅝ inches in diameter, and 12⅝ inches long, but whereas in the generator only five per cent. of the space is taken up by paper insulation, the space thus taken up in the motor amounts to 25 per cent.; in other words, we suppose there is 20 per cent. less iron in the motor armature. The winding of both armatures is the same, and consists of 780 turns of .657 ohm resistance. Each field magnet core is wound with 553 turns of wire, but the resistance of the coils in the generator is 1.53 ohms, that of the motor 1.44 ohms. the reduction in the resistance being due to the smaller diameter of the motor magnet cores.

"In the generator is also a shunt resistance of 20 ohms, arranged across the terminals of the field coils. This shunt is a coil of wire doubled back on itself so as to have no self-induction. By the addition of such a shunt any slight error in the design of the machines can be rectified, and a perfect agreement between their characteristics obtained; it is also said to increase the rapidity with which one machine corresponds to the other. If the load on the motor be suddenly diminished, the current will be as suddenly checked. By the adoption of the shunt, which acts as a kind of electro-magnetic damper or dash-pot, the kick from the magnet is principally taken up by the shunt wire, and the machines are thereby able to attain quickly their steady working condition."

A. L. Rohrer of the Thomson-Houston Electric company applied a modified plan for a large power plant by which 5,000 horse power is being transmitted a distance of twelve miles.

The diagram, Fig. 14, will serve to illustrate this idea. By this arrangement the generators are coupled mechanically in pairs as *one* unit on one shaft driven by *one* turbine, and electrically the armatures of each unit are connected in series.

Each armature has a potential of 2,500 volts. This gives 5,000 volts for each unit at the generating station. The generators are separately excited, and have also series winding, which will compensate for the loss in the line. At the receiving station there are the same number of units, each

consisting of two similar machines, with their armatures in series, and their fields separately excited, but without series winding. Each receiving unit is coupled to the same shaft in the same manner as the generating unit. At the generating station exciters only are used for charging the fields while at the receiving station, exciters are used in connection with a small storage battery which is necessary to start the first unit. The mechanically and electrically coupled units at the generating station are united electrically in parallel in one system by an equalizing bar, as shown in the diagram. It seems advisable to leave the storage battery in the circuit, permanently, to keep the fields of the motors fully excited, in case the speed should drop.

In another instance, 250 horse power is transmitted a

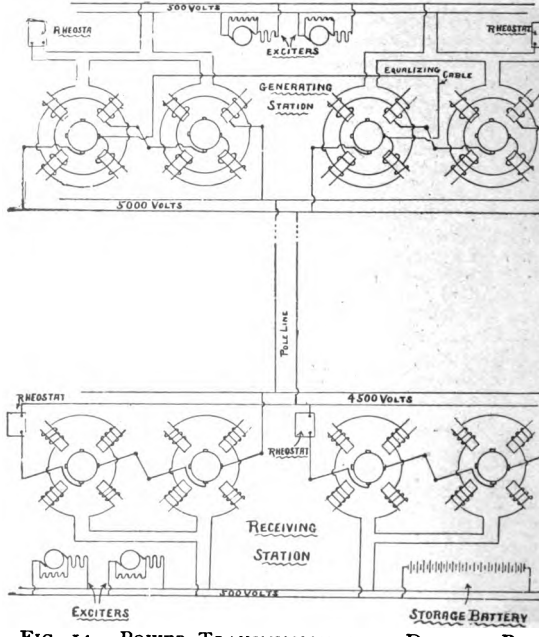


FIG. 14.—POWER TRANSMISSION WITH DYNAMO BATTERIES AT HIGH POTENTIAL.

distance of ten miles with single series wound units. Each machine is wound for a potential of 3,000 volts, and for this purpose a special commutator with 798 segments was constructed.

The plan of employing *single series wound units* for transmitting power from one point to another at high potentials is a most excellent one, as explained before.

d. The Shunt Motor on the Constant Potential System.

For the distribution of large and small power units over a larger area from a central station the shunt motor and the constant potential system are, generally speaking, the best combination. The magnetic field being independently excited, is constant and in a good shunt motor is so intense and the armature resistance so low that for most practical purposes the motor will run at a speed sufficiently uniform.

The electric generators for constant potential service are generally over-compounded to keep a constant pressure at the main distributing points. The system is the parallel system and motors are connected like incandescent lamps on the multiple arc system.

As a matter of fact a large number of shunt motors are in operation on isolated and central station direct current incandescent plants, the motors being wound for the same voltage as the lamps, which is generally about 110 volts.

On the three-wire incandescent system motors should be connected between the positive and negative wire and, therefore, be wound for 220 volts.

e. The Differential Motor on the Constant Potential System.

Where it is necessary to keep absolutely uniform speed this motor may be employed.

This motor will keep a uniform speed even when the electromotive force of the circuit varies within certain limits.

This mode of regulation is almost perfect. If, however, the load should be increased above the capacity of the motor or the electromotive force of the circuit fall too low the motor will still make a heroic effort to maintain a uniform speed and take more current than its armature wire can carry. This would blow out the safety catches, or in their absence would, of course, burn out the armature.

Generally a shunt motor will give a speed sufficiently uniform for most practical purposes, and should be preferably employed.

f. The Cumulative Motor on the Constant Potential Circuit.

This motor shows the characteristic features of both the series and shunt motor. It will run at its normal speed below a certain load. This load may be exceeded and the motor still perform its duty, but at a slower speed. In other words it will not do more than its rated work and will sacrifice time to weight. This motor is greatly used for freight elevator work If, for instance, the motor can raise the elevator weighing (with load) 10,000 pounds at the rate of 50 feet a minute and 20,000 pounds should be imposed on its good nature, it will lift the elevator, but only at the rate of about 25 feet a minute, of course doing a work of 500,000 foot pounds in each instance.

CHAPTER X.

Directions for Connecting and Operating Series Motors on Arc Light Circuits.

Set up the motor in the dryest, cleanest and best lighted place available, and insulate it from surrounding objects as perfectly as possible.

Connect it to the circuit as shown in the diagram, Fig. 15.

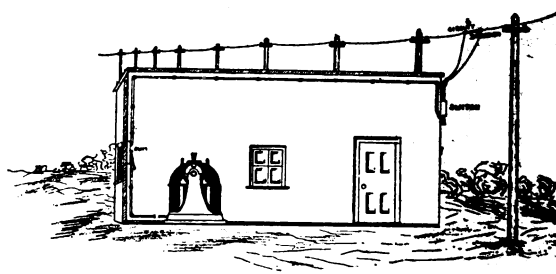


FIG. 15.—DIAGRAM SHOWING METHOD OF CONNECTING MOTORS TO ARC LIGHT CIRCUITS.

Transmit the power with a pliant belt of uniform thickness, and do not make the distance between the centers of motor and driven pulley too great, as the belt would flap;

nor too short, as the belt would not transmit the power except when excessively tight.

While working keep the belt just tight enough to drive properly by means of the slide screw, and slacken it when the motor stands idle.

Lubricate the bearings with a good oil which will not gum.

If the direction of rotation needs to be reversed, it is only necessary to reverse each brush holder on its pin, and change the "lead" to the opposite side.

Oil the commutator frequently, but sparingly by means of a cloth moistened with oil. If too much oil is applied there will be sparking, the oil short-circuiting the sections. If no oil is applied there will be excessive wear, by grinding of the commutator and brushes.

The brushes should not be set to bear too hard on the commutator.

When the wearing edge of the brush, which should never be allowed to exceed in width one segment of the commutator, grows too broad, turn the brush so that the narrow edge bears on the commutator.

Overloading the motor will lessen its power by lowering its speed.

In starting the motor, in order to prevent excessive strain, pull out the governor lever as far as possible thus cutting out the field coils, before opening the switch. After opening the switch quickly allow the lever to return slowly to its proper position, as the armature attains speed, making sure that the ball bearing on the shaft is in its socket in the lever.

To stop the motor close the switch, which cuts the motor out of circuit.

CHAPTER XI.

Directions for Connecting and Operating Shunt Motors on Constant Potential Circuits.

The motor is wound with the field magnets in shunt around the armature, and in the circuit from the armature is placed a variable resistance or rheostat. On starting the motor it is necessary that all this resistance be in circuit; otherwise, because of the low resistance of the armature, excessive current will flow, and damage may be done the

motor. This resistance is contained in a box on which a sliding arm moving over segments adjusts the current. Before starting the motor the arm should be against the stop in which position the circuit is open. After closing the switch the movable arm should be turned slowly, when the motor will gradually start up and set up a counter electromotive force which will act as additional resistance in the circuit. As the motor approaches full speed the arm should be turned as far as it will go, thus cutting all resistance out of circuit. It is necessary that the movement of the rheostat should be slow and regular, for if it is not, excessive current will flow, and the fusible plug will be melted.

The rheostat should never be used to regulate the speed of the motor, but merely for starting. Never allow the rheostat arm to rest on an intermediate segment.

To stop the motor, open the switch, and then turn the arm of the rheostat back to its first position.

Fig. 16 and Fig. 17 illustrate the mode of connecting the motor and rheostat to the constant potential mains.

The following summary may be found useful in directing those who operate motors:

Be sure the motor is well insulated from the earth.

Keep everything about the motor scrupulously neat and clean.

Always cover the motor when not running, with a dust proof cloth.

Never close the circuit until the brushes are arranged as nearly right as possible.

Read carefully the instructions received from the manufacturer regarding setting the brushes.

Remember that careless handling of brushes gives rise to more trouble than all other mistakes combined.

Never run too tight a belt; hot boxes are sure to follow.

Raise the brushes from the commutator on stopping the motor.

Never start the motor until oil is flowing from the oil cups.

Do not use too much oil on bearings. It does no good and makes a disagreeable mess.

Use nothing but the very best mineral oil for lubrication.

Do not expect the best results from an overloaded machine.

Never smooth the commutator with emery cloth. Sand paper may be used, but the commutators must be carefully examined to see that no copper bits project across the insulation so as to short-circuit any two segments.

Never attempt to do fifteen horse power work with a ten horse power motor. The motor may stand it for awhile, but the work will not be satisfactory, and in time damage will be done the motor.

Be sure that the wires carrying current to the motor are of ample size to give full pressure.

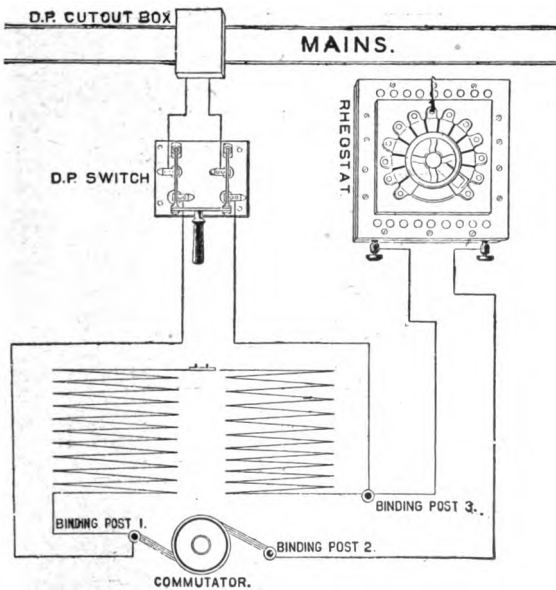


FIG. 16.—SHUNT MOTOR ON CONSTANT POTENTIAL CIRCUITS.

CHAPTER XII.

Rules and Formulæ for Ascertaining the Horse Power of Motors.

In ascertaining the horse power of a motor to drive various classes machinery, the following data will be found useful:

a. Hoisting in Vertical Shafts (Elevators).

If W=Weight to be lifted vertically in pounds per minute.
S=Speed per minute* at which the hoist is to run.
As one horse power is the power to raise 33,000 pounds

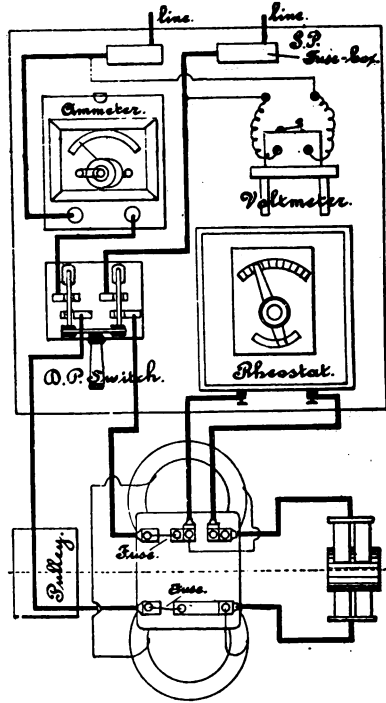


FIG. 17.—SHUNT MOTOR ON CONSTANT POTENTIAL CIRCUIT.

one foot high, in one minute, the horse power required to run the hoist will be: $H.P.=\frac{W \times S}{33,000}$

*NOTE.—Compare Table XVII. If speed is given in miles per hour, or feet per second, reduce to feet per minute.

This, however, does not allow for loss in friction. From practical experiments it appears that the loss amounts to from 40% to 65%; spur gearing being more economical than worm gearing. The above value should therefore be increased. Assuming a loss of 50%, or an efficiency of 50%, the above value should be multiplied by $\frac{100}{50}=2$, which gives us the formula:

$$\text{H.P. of motor}=\frac{W\times S\times 2}{33{,}000}=\frac{W\times S}{16{,}500}$$

EXAMPLE: Lift 900 pounds 80 feet a minute, the cage being counter-balanced (preferably on the drum):

$$\text{H. P. of motor}=\frac{900\times 80}{16{,}500}=4.36$$

or about 4½ horse power.

b. Hoisting in Inclined Shafts.

If W=Weight in pounds per minute to be raised on incline.
S=Speed per minute at which cage (or skip) is to be raised.
Θ=The inclination of the shaft.

Again, assuming the cage to be counterbalanced it will, of course, require less power to lift the same weight up an incline than vertically, the smaller the angle (*i e.*, the nearer the incline is to the horizontal), the less power it will take.

We found for vertical hoisting the formula:

$$\text{H P.}=\frac{W\times S}{16{,}500}$$

For an incline it will be approximately:

$$\text{H.P.}=\frac{W\times S\times \text{sine }\Theta}{16{,}500}$$

EXAMPLE: Lift 900 pounds 80 feet a minute, cage being counterbalanced, on an incl ne of 45° * (100%, or a rise of 100 feet for each 100 feet horizontal distance); natural sine of 45°=.7. (See Table XX).

$$\text{H.P. of motor}=\frac{900\times 80\times .7}{16{,}500}=3.05 \text{ or about 3 H.P.}$$

c. Pumping Water.

One cubic foot of fresh water weighs 1000 ounces or

*NOTE.—"*Percentage of Grade*" is very often wrongly defined. One per cent. grade means one foot rise to 100 feet distance. measured on a *horizontal* line. In other words, the percentage is the *tangent* of the angle of the grade and *not the sine.*

$\frac{1000}{16}$=62.5 pounds; one cubic foot contains 7.5 gallons; hence weight of one gallon $\frac{62.5}{7.5}$=8.33 pounds.

As one square foot contains 144 square inches, a column of water 1 foot high and 1 square inch in cross-section weighs $\frac{62.5}{144}$=.434 pounds.

If we now call the height in feet to which the water is to be lifted H, and the pressure in pounds per square inch P, we find:

$$P=.434\ H$$

$$H=\frac{P}{.434}=2.3\ P$$

If G=number of gallons *per minute* to be pumped, we find:

$$H.P.=\frac{G\times H\times 8.33}{33000}$$

This formula, however, does not allow for water friction and for loss in gearing.

Assuming these losses on an average as 34% (or an efficiency of 66%)*, we get:

H.P. of motor $=\frac{G\times H\times 8.33}{33000\times .66}=\frac{G\times H}{2614}$ or in round figures

H P of motor $=\frac{G\times H}{2600}$

EXAMPLE: To lift 24000 gallons per hour 650 feet.

Solution: 24000 gallons per hour is $\frac{24000}{60}$= 400 gallons per minute.

Hence: H.P. of motor $=\frac{400\times 650}{2600}$=100 H.P.

d. Pumping against Pressure in Tanks.

If we have to pump against pressure in tank (as, for instance, in elevator work), we substitute P for H in our formulæ; as H=2.3 P, we get:

$$H.P.=\frac{G\times 8.33\times 2.3\times P}{33000}$$

* NOTE.—An efficiency of from 60% to 66% is obtained only under favorable conditions. The efficiency will greatly depend upon the length of the line through which water is to be forced, the number of turns, etc., and the size of pipe. These are points which must be considered in each case separately.

or, $$H.P.=\frac{G\times P}{1722}$$

which is the *theoretical* number of horse power not taking into consideration the losses from friction; taking the formula

$$H.P. \text{ of motor}=\frac{G\times H}{2600}$$ and substituting 2.3 P for H we get:

$$H.P. \text{ of motor}=\frac{G\times 2.3\times P}{2600}$$

or, $$H.P. \text{ of motor}=\frac{G\times P}{1130}$$

Which is a formula allowing for losses from friction.

EXAMPLE: An elevator uses 300 gallons per minute; pressure required in closed tank 100 pounds.

Solution: Horse power of motor $=\frac{300\times 100}{1130}=26.5$ horse power.

e. Ventilation.

Below are given the speeds at which a certain make of fan runs, and the actual horse power required. To the horse power should be added at least fifty per cent. for losses from belting and friction. If the air is to be drawn through pipes the power required is greater and should be carefully determined by a competent engineer.

TABLE XVI.

SPEED, HORSE POWER USED, AND AMOUNT OF AIR EXHAUSTED.

Size.	Revolutions per Minute.	Horse Power Used.	Exhaust Cubic Feet of Air per Minute.
12 in.	1,000	¼	1,500
18 in.	700	½	3,000
24 in.	600	¾	4,500
30 in.	500	1	7,500
36 in.	400	2	12,000
48 in.	400	4½	26,000
54 in.	400	5	32,000
60 in.	400	5½	42,000
72 in.	300	5¾	45,000
84 in.	250	8	56,000
96 in.	200	9	63,000

NOTE.—The power required varies as the cube of the speed. For example, if the speed is doubled eight times as much power will be required.

f. Electric Traction.

TABLE XVII.

MILES PER HOUR, IN FEET PER MINUTE AND PER SECOND.

Miles per hour.	Feet per Minute.	Feet per Second.	Miles per Hour.	Feet per Minute.	Feet per Second.	Miles per Hour.	Feet per Minute.	Feet per Second.
1	88	1.46	11	968	16.13	21	1,848	30.8
2	176	2.93	12	1,056	17.6	22	1,936	32.26
3	264	4.4	13	1,144	19.07	23	2,024	33.72
4	352	5.87	14	1,232	20.52	24	2,112	35.2
5	440	7.33	15	1,320	22.	25	2,200	36.67
6	528	8.8	16	1,408	23.47	26	2,288	38.14
7	616	10.26	17	1,496	24.93	27	2,376	39.6
8	704	11.73	18	1,584	26.4	28	2,464	41.04
9	792	13.2	19	1,672	27.86	29	2,552	42.50
10	880	14.67	20	1,760	29.33	30	2,640	44.0

FORMULA FOR CALCULATING THE HORSE POWER REQUIRED TO PROPEL A VEHICLE.

$$H.P. = \frac{WKS}{33000} + \frac{W2240S}{33000} \sin.$$

where W=total rolling load in tons,
K=resistance to motion expressed in lbs. per ton,
S=speed required in feet per minute,
θ=the inclination of the grade.

The value of S corresponding to any number of miles per hour can be filled in from the table. K can only be determined by experiment, but approximate values can be found in the usual text books. For raised rails K varies from 12.5 lbs. to 20 lbs ; for sunk rails, from 28 lbs. to 56 lbs. will be fair limits under ordinary working conditions. On inspection it will be seen that the first fraction in the equation gives the horse power required to overcome the resistance to traction on the level only, while the right hand quantity is a measure of the work done either against or by gravity; hence the double sign. The term will be negative on a down-grade, and positive on an up grade. If the *gravity* term exceeds the *traction* term and have a negative sign, the equation becomes negative. This means that the car will travel by the aid of gravity alone. The use of the table and formula will best be seen from an example.

Let K =30, W=8 tons, Speed=10 miles per hour,
$\therefore$ S=880.

And, let $\sin\theta=\frac{1}{50}$; or, $\theta=1°9'$.

Suppose the gradient is against the load, we have

$$\text{H.P.}=\frac{8\times30\times880}{33000}+\frac{8\times2240\times880\times1}{33000\times50}=6.4+9.5=15.9.$$

TABLE XVIII.

TONS PER H. P. ON T RAILS.

K = 16 LBS.

% Grade.	Miles per Hour.	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	1	23.43	9.768	6.168	4.509	3.554	2.939	2.497	2.173	1.883	1.730	1.534	1.437	1.326	1.230	1.150	1.074	1.014	.9581	.9084	.8638	.8235
	2	11.71	4.884	3.084	2.254	1.777	1 469	1.248	1.086	.941	.865	.767	.718	.663	.615	.575	.537	.507	.479	.454	.432	.411
	3	7.81	3.256	2.056	1.503	1.184	.979	.832	.724	.627	.576	.511	.479	.442	.410	.383	.358	.338	.319	.302	.287	.274
	4	5.85	2.442	1.542	1.127	.888	.734	.624	543	.470	.432	.383	.359	.331	.307	.287	.268	.253	.239	.227	.216	.205
	5	4.68	1.953	1.233	.902	711	.588	.499	.434	.376	.346	.307	.287	.265	.246	.230	.215	.202	.191	.181	.172	164
	6	3.90	1.628	1 028	.751	.592	.489	.416	.362	.313	.288	.255	.239	.221	.205	.191	.179	.169	.159	.151	.143	137
	7	3.34	1.395	.881	.644	508	419	357	31	.269	.247	.219	.205	.189	.175	164	.153	.145	.137	.129	123	.117
	8	2.92	1 221	.771	.563	.444	.367	.312	.271	.235	.216	.191	.179	.165	.153	.143	.134	.126	.119	.113	.108	.102
	9	2.60	1.085	.685	.501	.394	.326	.277	.241	.209	.192	.170	.159	.147	.136	.127	.119	.112	.106	.100	.095	.091
	10	1.34	.9768	.617	.451	.355	.294	.249	.217	.188	.173	.153	.143	.132	.123	.115	.107	.101	096	.091	.086	.082
	12	1.95	.814	.501	.375	.296	.244	.208	.181	.156	.144	.121	.119	.1.0	.102	.095	.089	.084	.079	.075	.071	.068
	15	1.56	.651	.411	300	.237	.196	.166	.145	.125	.115	.102	.095	.088	.082	.076	.071	.067	.063	.060	.057	.054
	20	1.171	.488	.308	.225	.177	.147	.125	.108	.094	.86	.076	.072	.066	.061	.057	.053	.050	.048	.045	.043	041
	25	.936	.390	.246	.180	.142	.117	099	.087	.075	.069	.061	.057	.053	.049	.046	.043	.040	038	.036	.034	.032
	30	.781	.325	.205	.150	.118	.098	.083	.072	.062	.057	.051	.048	.044	.041	.038	.036	.034	032	.030	.028	.027
% Grade.		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Angle.			34′ 22″	1° 8′ 45″	1° 43′ 6″	2° 17′ 26″	2° 51′ 19″	3° 26′	4° 0′ 28″	4° 39′ 24″	5° 8′ 30″	5° 42′ 30″	6° 16′ 40″	6° 50′ 3″	7° 24′ 20″	7° 58′ 10″	8° 31′ 50″	9° 5′ 20″	9° 38′ 50″	10° 12′ 10″	10° 45′ 30″	11° 18′ 40″

TABLE XIX.

TONS PER H. P. ON GIRDER RAILS.

K = 40 LBS.

% Grade.	Miles per Hour.	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	1	9.375	6 0102	4.422	3.499	2.895	2.474	2.153	1.908	1.715	1.557	1.427	1.316	1.222	1.141	1 070	1.007	.9521	.9028	.8585	.8234	.7823
	2	4.687	3.005	2 211	1.749	1.447	1.237	1.076	.954	.857	.778	.713	.658	.611	.570	.535	.503	.476	.451	.4292	.411	.3911
	3	3.125	2.003	1.474	1.166	.965	.824	.717	.636	.571	.519	.475	.439	.407	.380	.357	.336	.317	.301	.286	.274	.2607
	4	2 343	1.502	1.105	.874	.723	.618	.538	.477	.428	.389	.356	.329	.305	.285	.267	.251	.238	.225	.214	.205	.195
	5	1.875	1.202	.884	.699	.579	.495	.430	.381	.343	.311	.285	.263	.244	.228	.214	.201	.190	.180	.171	.164	.156
	6	1 562	1.001	.737	.583	.482	.412	.358	.313	.285	.259	.237	.219	.203	.190	.178	.168	.158	.150	.143	.137	.130
	7	1 339	.858	.631	.499	413	.353	.302	.272	.245	.222	.204	.188	.174	.163	.152	.144	.136	129	122	117	.111
	8	1 171	.751	.552	.437	.361	.301	.269	.238	.214	.194	.178	.164	.152	.142	.133	.125	.119	.112	.107	.102	.097
	9	1.041	.667	.491	.388	.321	.274	.239	.212	.190	.173	.158	.146	.135	.126	.119	.112	.106	.100	.095	.091	.086
	10	.937	601	.442	.350	.289	.247	.215	.191	.171	.155	.142	131	.122	.114	.107	.100	.095	090	086	.082	.078
	12	.781	.500	.363	.291	.241	.206	.179	.150	.142	.129	.118	.109	101	.095	.089	.084	.079	.075	071	068	065
	I5	.625	.400	294	.233	.193	.165	.143	.127	.114	.103	.095	.087	.081	.076	.071	.067	.063	.060	.057	.054	.052
	20	.468	.300	.221	.175	.144	.123	.107	.095	.085	.078	.071	.066	.061	.057	.053	.050	047	045	.043	.041	.039
	25	.375	.240	.177	.139	.116	.099	.086	.076	.068	.062	.057	.052	.049	.045	.042	.040	.038	036	034	.032	.031
	30	.312	.200	.147	.116	.096	.082	.071	.063	.057	.052	.047	.044	.040	.038	.035	.033	.031	.022	028	.027	026
% Grade.		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Angle.			34′ 22″	1° 8′ 45″	1° 43′ 6″	2° 17′ 26″	2° 51′ 19″	3° 26′	4° 0′ 28″	4° 34′ 24″	5° 8′ 30″	5° 12′ 30″	6° 16′ 40″	6° 50′ 30″	7° 24′ 20″	7° 58′ 1.0″	8° 31′ 50″	9° 5′ 20″	9° 38′ 50″	10° 12′ 10″	10° 45′ 30″	11° 18′ 40″

TABLE XX.

PER CENT. GRADE AND SINES OF ANGLE OF INCLINED PLANE.

% GRADE.	Angle	NATURAL SINE.	% GRADE.	Angle	NATURAL SINE.
0 000	0	0.00000	100.000	45	0 70711
1.746	1	0 01745	108.553	46	0.71934
3.492	2	0.03490	107.237	47	0.73135
5.241	3	0.05234	111.061	48	0 74314
6 998	4	0.06976	115 037	49	0.75471
8.749	5	0.08716	119.175	50	0.76604
10.510	6	0.10453	123.490	51	0.77715
12.278	7	0.12187	127.994	52	0.78801
14.054	8	0.13917	132.704	53	0.79864
15 838	9	0 15643	137.638	54	0.80902
17.633	10	0.17365	142.815	55	0.81915
19 438	11	0.19081	148.256	56	0.82904
21.256	12	0.20791	153 986	57	0.83867
23.087	13	0.22495	160.033	58	0.84805
24.933	14	0 24192	166 428	59	0.85717
26.795	15	0.25882	173.205	60	0 86603
28 675	16	0.27564	180.405	61	0 87462
30.573	17	0.29237	188.073	62	0 88295
32.492	18	0.30902	196.261	63	0.89101
34 433	19	0.32557	205.030	64	0.898:9
36 397	20	0.34202	214.451	65	0.90631
38.386	21	0.35837	224 604	66	0.91355
40.403	22	0.37461	235.585	67	0.92050
42.447	23	0.39073	247.509	68	0.92718
44.523	24	0.40674	260.509	69	0.93358
46.631	25	0.42262	274.748	70	0.93969
48.773	26	0.43837	290.421	71	0 94552
50.953	27	0.45399	307.768	72	0.95106
53 171	28	0.46947	327.085	73	0 95630
55.431	29	0.48481	348.741	74	0.96126
57 735	30	0 5000)	373.205	75	0.96593
60 086	31	0.51504	401 1078	76	0 97030
62 487	32	0.52992	433.148	77	0 97437
64.941	33	0.54464	470.463	78	0.97815
67.451	34	0.55919	514.455	79	0.98163
70 021	35	0.57358	567.128	80	0.98481
72 654	36	0.58779	631 375	81	0.98769
75.355	37	0.60182	711 537	82	0.99027
78.129	38	0 61566	814.435	83	0.99255
80.978	39	0.62932	951.436	84	0 99452
83.910	40	0.64279	1143.01	85	0 99619
86 929	41	0.65606	1430.07	86	0.99756
90.040	42	0.66913	1908.11	87	0.99863
93.252	43	0.68200	2863.63	88	0 99939
96.569	44	0.69466	5729.00	89	0.99985
			Infinity	90	1.00000

CHAPTER XIII.

Pulleys, Belts, Shafting.

PULLEYS.

It is not desirable to use a larger diameter pulley than that sent with the generator or motor.

A smaller pulley is objectionable because to obtain necessary friction and belt contact increased tightening of the belt will be necessary. This is liable to result in heating of the bearing.

Where smaller diameter of pulley is unavoidable, the width of belt and face of pulley should be increased from 25 to 50 per cent. This will tend to give additional belt contact, and admit of running a slack belt. The belt should be a little narrower than the face of the pulley.

BELTS.

Belts should be perfectly straight and of equal thickness throughout. They should be made endless. Where lacing is unavoidable, the ends to be laced should be cut at right angles to the sides. The lace holes should be formed by an oval punch, and extend lengthwise, thus reducing the section the least. Commencing at the middle of the belt, lace evenly so as to give uniform strength, and never cross the lace on the inside of the belt. If copper or other rivets are used, the heads should be rather below the level of the inside surface of the belt, to prevent contact with the pulley.

Belts and pulleys should be kept clean and free from accumulations of dust and grease, and particularly from contact with lubricating oils, some of which permanently injure leather. Tallow applied blood warm will render leather belts pliable.

Both short and vertical belts are disadvantageous. They always require more tension than long or horizontal ones. As the transmitting power of a belt increases greatly with an increase of arc of contact, the slack side should always be on top.

Belts are less liable to slip and adhere better at quick speeds. Quick speed belts should, if possible, have no laced joint. A moderately slack belt gives best satisfaction; it lasts longer, and heats the bearings least. Tightening pulleys should be avoided; when they must be used they should be of large diameter and on the slack side of the belt. The most effective tightener is the weight of the belt on its slack side, which increases adhesion by increas-

ing circumferential contact with the pulley. The adhesion will be decreased if the pulley face is too convex.

For ordinary light work single leather belts are more satisfactory. The flesh side of a leather belt is stronger and least liable to crack; hence it should be on the outside. Moreover, the smooth side of the belt lies closer to the pulley without air cushion. Small pulleys injure belts by too much flexure. A belt will not carry as much when new as it will after a few months' careful use. Leather belting may be calculated to have a tensile strength of three hundred and fifty pounds per square inch of section.

The width of a belt depends upon the tension, the size of the smaller pulley and the portion embraced by the belt and the speed of belt. The width may be roughly calculated as follows:

Single Belt. $$W=\frac{H.\ P.\times 3000}{D\times R}$$

Double Belt. $$W=\frac{H.P\times 2000}{D\times R}$$

W=Width in inches.
H. P=Horse power to be transmitted.
D=Diameter in inches of driven pulley.
R=Revolutions per minute of driven pulley.

TABLE XXI.

FOR SINGLE LEATHER BELTING.

1 inch wide, 800 feet per minute=1 Horse Power.

Speed in feet, per minute.	WIDTH OF BELTS IN INCHES.											
	2	3	4	5	6	8	10	12	14	16	18	20
	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H.P.	H P.
400	1	1½	2	2½	3	4	5	6	7	8	9	10
600	1½	2¼	3	3¾	4½	6	7½	9	10½	12	13½	15
800	2½	3	4	5	6	8	10	12	14	16	18	20
1000	2	3¾	5	6¼	7½	10	12½	15	17½	20	22½	25
1200	3	4½	6	7½	9	12	15	18	21	24	27	30
1500	3¾	5¾	7½	9½	11½	15	18¾	22½	26½	30	33¾	37½
1800	4½	6¾	9	11¼	13½	18	22½	27	31½	36	40½	45
2000	5	7½	10	12½	15	20	25	30	35	40	45	50
2400	6	9	12	15	18	24	30	36	42	48	54	60
2800	7	10½	14	17½	21	28	35	42	49	56	63	70
3000	7½	11¼	15	18¾	22½	30	37½	45	52½	60	67½	75
3500	8¾	13	17½	22	26	35	44	52½	61	70	79	88
4000	10	15	20	25	30	40	50	60	70	80	90	100
4500	11¼	17	22½	28	34	45	57	69	78	90	102	114
5000	12½	19	25	31	37½	50	62½	75	87½	100	112	125

Double leather belting will transmit about 50 per cent. more power than is shown in this table.

When it is not convenient to measure the length of belt over driving and driven pulley with the tape line, apply the following rule:

$L = 1.62 \times (D + d) + 2F$, where:

L—Length of belt in feet.

D—Diameter of driving pulley in feet.

d—Diameter of driven pulley in feet.

F—Distance betwen centers of shafts in feet.

SHAFTING.

The *torsional strength of shafts* or their resistance to breaking by twisting is proportional to the cube of their diameter. Their *stiffness* or resistance to bending, is proportional to the fourth power of their diameters, and varies inversely as their load and inversely as the cube of the length of their spans between bearings or "bay."

Hence the diameter of a shaft must be determined (for the same number of horse power to be transmitted) according to whether it is:

a. The head shaft carrying main driving pulley or gear well supported by bearings, or

b. The line shafting with bearings about 8 feet apart, or

c. The shafting for simply transmitting power. Short counter shafts.

The following table gives the transmitting capacity of these three classes for cold rolled and turned iron shafting for *one revolution per minute.* The figures in the second and third vertical columns must be multiplied by the number of revolutions the shaft is making per minute in order to get the number of horse power it can transmit.

EXAMPLE 1.—What is the transmitting capacity of a countershaft of cold rolled iron 2" diameter at 300 revolutions per minute?

Solution.—Table *c* gives for one revolution .26 horse power. Hence for 300 revolutions we get:

$$300 \times .26 = 78 \text{ H. P.}$$

EXAMPLE 2.—A turned iron line shaft shall run at 150 revolutions and transmit 44 horse power. What should be its diameter?

Solution.—The horse power of a certain size shaft in these tables of course equals horse power per one revolution multiplied by the number of revolutions per minute.

or H. P.—H. $P._1 \times$ rev.

TABLE XXII.

TRANSMITTING CAPACITY OF IRON SHAFTING

Diameter of Shaft. Inches.	Cold Rolled.	Turned.
	H. P. for one revolution per minute.	
a. Head Shafts.		
1½	.045	....
1¾	.071	.043
2	.107	.064
2¼	.15	.081
2½	.21	.125
2¾	.27	.16
3	.36	.20
3¼	.45	.27
3½	.57	.34
3¾	.70	.42
4	.85	.51
4½	1.21	.72
5	....	1.00
5½	....	1.33
b. Line Shafts.		
1⅛	.067	....
1⅜	.086	....
1¾	.107	.060
1⅞	.132	.073
2	.16	.089
2⅛	.19	.106
2¼	.22	.126
2¾	.27	.15
2½	.31	.17
2¾	.41	.23
3	.54	.30
3¼	.68	.38
3½	.85	.47
3¾	....	.58
4		.71
c. Countershafts.		
1¼	.065	....
1⅜	.085	....
1½	.112	.067
1⅝	.142	.086
1¾	.18	.107
1⅞	.22	.132
2	.26	.16
2⅛	.32	.19
2¼	.38	.22
2⅜	.44	.27
2½	.52	.31
2¾	.69	.41
3	.90	.54
3¼	....	.68
3½	....	.85

Hence H. $P._1 = \frac{H.P.}{rev.}$

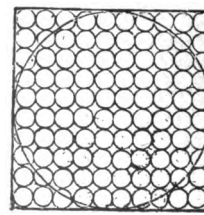
Hence H. $P._1 = \frac{44}{150} = .29$

We find under "*turned shafting*" in Table *b* the next higher number, .30, which corresponds to a *three-inch shaft.*

CHAPTER XIV.

Wire Gauges.

There is a multiplicity of wire gauges. For electric light and power circuits, however, only two are used in the United States, viz.: The Brown & Sharpe or American gauge and the Birmingham gauge. Wires are generally



The figure is 100 times actual size.

FIG. 18.*—CIRCULAR MILS.

"gauged" by measuring their diameters. What we really want to know, however, is the sectional area of wires, which varies in the ratio of the square of the diameter. A thousandth part of an inch, called a "mil," is usually taken as the unit of measure of the diameter of a wire.

Since wires are round, and the areas of circles increase as the squares of the diameters, we may regard the "mil" as unit circular wire.

By squaring the diameter "d" in "mils" of any wire, we obtain at once its area in "circular mils," that is to say, the number of unit wires to which it is equivalent.

This is illustrated in the figure above, where the large circle represents a wire 10 mils in diameter, and the smal

*The author is indebted to Joseph W. Marsh, general manager Standard Underground Cable company, for the use of this illustration, which was taken from No. XI Pocket Hand-book presented by that company.

circles are the circular mils (100) whose united areas equal the area of the large circle.

As the area of a circle is $\frac{d^2 \times 3.1416}{4}$ we must multiply the number of *circular* mils by $\frac{3.1416}{4}$ or .7854 in order to get the cross-sectional area in *square* mils.

Thus the 100 circular mils in Fig. 18 equal $.7854 \times 100 = 78.54$ *square* mils. As a square inch equals 1,000,000 square mils, 78.54 square mils $= \frac{78.54}{1,000,000} = .00007854$ square inches.

The following data show the relation between square and circular mils.

Symbols: CM (or M)=Circular mils.
Sq. m.=Square mils.
1 CM.=.7854 sq. m.
1 sq. m.=1.2732 CM.
1 sq. inch=1,000,000 sq. m.
1 sq. inch=1,273,200 CM.
1 sq. inch=area of a circle of 1.128 inch diameter.

Area of a circle 1 inch (1000 mils) diameter=1,000,000 CM=785,400 sq. m.

In Tables XXIII, XXIV and XXV are given the circular mils (and other data) for the B. & S. and B. W. G., and for the English Board of Trade standard gauges.* In ordering special sizes of wire, it is always advisable to give either the diameter of the wire in mils or the cross-section in circular mils, not the gauge.

One mistake often made is to call, for instance, "0000" wire "four 0" wire. This is entirely wrong.

Table XXIII shows that in the B. & S. gauge a "0000" wire is equal to 211,600 circular mils, while the term "four 0" wire indicates a conductor made up of four single "0" wires equal to $105,593 \times 4 = 422,372$ circular mils.

Fig. 19 shows the micrometer gauge which measures the diameter of a wire in mils, and will answer in every case, no matter which gauge may be used by the manufacturer.

The operation of measuring a wire with the micrometer gauge is as follows:

* The last mentioned table was added for the convenience of our English readers.

The wire to be measured is placed between a fixed support B, and the end C of a long. movable screw, which accurately fits a threaded tube a. A thimble D, provided with a milled head fits over the screw C, and is attached to the upper part. The lower circumference of D is divided into a scale of 25 equal parts, each representing $\frac{1}{1000}$ of an inch or 1 mil.

The tube a is graduated into larger divisions which equal $\frac{1}{10}$ of an inch or 100 mils. Each of these larger divisions is divided into four subdivisions, each representing $\frac{1}{40}$ or $\frac{25}{1000}$ of an inch, or 25 mils

Suppose, now, a wire is placed between B and C, and the screw advanced until it fairly fills the space between

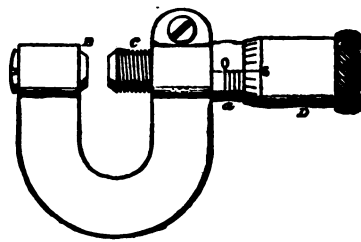


FIG. 19.—MICROMETER GAUGE.

them, and the reading shows one of the larger divisions on the scale a, two of the smaller ones on the same scale, and five on the end of the sleeve D. Then

One larger division of scale a.....	=100 mils
Two smaller divisions of scale a.....	= 50 "
Five divisions on circular scale on D.....	= 5 "
Diameter of wire.....	=155 mils

The circular scale on D bears the figure 25 along side of the zero mark o. This figure 25 indicates that the circular scale is divided into twenty-five equal parts.

Some manufacturers divide this circular scale into twenty equal parts, and the larger division on tube a into five subdivisions. In this case the above readings would give this result:

One larger division of scale a.....	=100 mils
Two smaller divisions of scale a.....	= 40 "
Five divisions on circular scale on D.....	= 5 "
Diameter of wire.....	=145 mils

The wire tables on pages 86 to 88 are calculated on the following basis:

1. One cubic foot of copper weighs 555 lbs.; hence one cubic inch weighs .3211896 lbs., and one mil-foot weighs .000003027149 lbs.

2. Resistance of one mil-foot of soft drawn copper of 98% conductivity at 32°C (89.6° F)=10.74 *true* ohms. (Correctly 10.73917.)

According to the latest researches the *true* ohm is represented by the resistance of a column of pure mercury, 1 square millimeter in section, and 106.3 centimeter long at 0°C. and one B. A. unit equals .9866 *true* ohm.*

$$\text{Hence:} \quad \text{R. 1000 ft.} = \frac{10740}{d^2}$$

3. The column "Safe Carrying Capacity" allows for *bare overhead* wires a temperature increase of 40°C (72°F), and for *insulated house* wires a temperature increase of 10°C (18°F.). The figures are calculated from the formulæ:

$$\text{Current} = \left(\frac{d^2}{40}\right)^{3/4}$$ for bare wire (72°F temperature increase).

$$\text{Current} = \left(\frac{d^2}{216}\right)^{3/4}$$ for insulated house wire (18°F temperature increase).

These formulæ were derived from A. E. Kennelly's formulæ and experiments.

*See report of the committee on electrical standards, read before the British Association, Leeds, September, 1890, and *Western Electrician*, Feb. 21, 1891, page 91.

TABLE XXIII.

BROWN & SHARPE WIRE GAUGE.

Gauge Number. B. & S	Diameter in Mils = 1/1000 inch. d	Sectional Area in Circular Mils. d^2 = C. M.	Weight of Bare Copper Wire per 1,000 feet. lbs.	Resistance of Copper Wire of 98% Conductivity at 89.6° F. 1,000 ft. in Ohms.	Safe Carrying Capacity in Amperes. Bare Overhead Wire.	Insulated House Wire.
0000	460	211600	640.5	.050756	620	175
000	410	167805	508.0	.064004	525	147
00	365	133079	402.8	.080704	438	124
0	325	105593	319.6	.101712	369	104
1	289	83695	253.4	.128318	309	87
2	258	66373	201.0	.161812	260	73
3	229	52634	159.3	.204040	219	62
4	204	41743	126.4	.257291	183	49
5	182	33102	100.2	.324441	154	44
6	162	26251	79.46	.409136	130	37
7	144	20817	63 01	.515932	109	31
8	128	16510	49.98	.650526	92	26
9	114	13094	39 64	.820222	77	22
10	102	10382	31.43	1.034580	65	18
11	91	8234	24.93	1 304340	54	15.4
12	81	6530	19 77	1.644740	46	12.9
13	72	5178	15.68	2.074000	38	10.8
14	64	4107	12 43	2.68)440	32	8 9
15	57	3257	9.86	3.297820	27	7.6
16	51	2583	7.82	4.158120	23	6.4
17	45	2048	6.20	5.243630	19	5.4
18	40	1624	4 92	6.61208)	16	4 5
Mil	1	1	.003027149	10740.	.0629	.0177

TABLE XXIV.

BIRMINGHAM WIRE GAUGE.

Gauge Number. B. W. G	Diameter in Mils = 1/1000 inch. d	Sectional Area in Circular Mils. d^2=C. M.	Weight of Bare Copper Wire per 1000 feet. lbs.	Resistance of Copper Wire of 98% conductivity at 89.6° F. 1000 ft. in Ohms.	Safe Carrying Capacity in Amperes. Bare Overhead Wire.	Insulated House Wire.
0000	454	206116	623 9	.52107	608	172
0 0	425	18 0625	546 8	.59460	551	155
00	380	144400	437.1	74377	466	132
0	340	115600	349.9	.92907	394	111
1	300	90000	272.4	.11933	327	92
2	284	80656	244.2	.13316	301	85
3	259	67081	203.0	.16011	263	74
4	238	56644	171.5	.18961	231	65
5	220	48400	146.5	.22190	205	58
6	203	41209	124.7	.26062	182	51
7	180	32900	98.08	.33148	152	43
8	165	27225	82.41	.39482	133	38
9	148	21904	66.31	.49032	113	32
10	134	17956	54.35	59813	97	28
11	120	14400	43.59	.74583	83	23
12	109	11881	35 96	.90396	72	20
13	95	9025	27.32	.11900	58	16
14	83	6889	20.85	15590	48	13 4
15	72	5184	15 69	.20718	39	10.8
16	65	4225	12.79	.25420	33	9.3
17	58	3364	10.18	.319263	28	7.8
18	49	2401	7 268	.44734	22	6.1
Mil	1	1	.003027 49	10740.	.0629	.0177

TABLE XXV.

ENGLISH BOARD OF TRADE STANDARD GAUGE.

Gauge Number. B. T. W.	Diameter in Mils = 1/1000 inch. d.	Sectional Area in Circular Mils. d^2 = C. M.	Weight of Bare Copper Wire. Per 1000 ft. lbs.	Resistance of Copper Wire of 98% Conductivity, at 89 6° F. 1,000 ft. in Ohms.	Safe Carrying Capacity Amperes. Bare Overhead Wire.	Safe Carrying Capacity Amperes. Insulated House Wire.
0000000	500	250000	756.8	.042996	703	199
000000	464	215296	651.7	.049485	628	177
00000	432	186624	564.9	.057549	564	159
0000	400	160000	484.4	.06713	503	142
000	372	138384	418.9	.07761	450	127
00	348	121104	366.6	.08868	408	115
0	324	104976	317.8	.10231	367	103
1	300	90000	272.4	.11933	327	92
2	276	76176	230.6	.14099	288	82
3	252	63504	192.2	.16912	251	71
4	232	53824	162 9	.19954	222	63
5	212	44944	136.1	.23896	183	55
6	192	36864	111 6	.29134	167	47
7	176	30976	93.76	.34672	147	41
8	160	25600	77.50	.41953	127	36
9	144	20736	62 77	.51794	108	31
10	128	16384	49.60	.65552	91	26
11	116	13456	40 73	.79816	79	22
12	104	10816	32.74	.99297	67	19
13	92	8464	25.62	1.26897	56	16
14	80	6400	19.37	1 67813	45	12 7
15	72	5184	15.69	2.07176	38	10 8
16	64	4096	12 40	2 62207	32	9.1
17	56	3136	9.493	3.42487	26	7.4
18	48	2304	6.975	4 66154	21	5.9
Mil.	1	1	.003027149	10740.	.0629	.0177

NOTE.—This Gauge is not used in the United States.

CHAPTER XV.

Recapitulation of Important Electric Transmission Formulæ and Definitions of Electrical Units.

SYMBOLS USED IN TABLE XXVI.

C=Current in amperes for N mechanical horse power delivered by motor shaft.

E=E. M. F. at terminals of motor.

V=Number of volts lost in conductor.

N=Number of mechanical horse power delivered by motor shaft.

M=Circular mils.

D=Distance in feet (one way) plus 5% for sag.

l=Commercial efficiency of electric system.

a= " " " motor.

b= " " " generator.

Written as decimal fractions.

%=Per cent. of electrical energy lost in conductor.

K=Cost in cents per lb. of bare copper wire.

G=Cost in dollars of generator per electrical horse power delivered at generator terminals (including cost of freight, setting, foundations, electrical instruments and connecting up.)

In formulæ for minimum cost of plant we assumed:

a=.90 (commercial efficiency of motor)

b=.90 (" " " generator.)

K=25 (cost in cents of *bare* copper wire per lb. delivered at the poles.)

G=45 (cost in dollars of generator set up, per electrical horse power delivered at its terminals.)

P=25 (cost in dollars of power (water) set up, per mechanical horse power, delivered at generator pulleys).

TABLE XXVI.

RECAPITULATION OF IMPORTANT ELECTRIC TRANSMISSION FORMULÆ.

DATA RELATING TO:		CROSS-SECTIONAL AREA.		WEIGHT.		COST.	
Conductor... (Bare Copper.)	As determined by amount of energy lost therein.	3	$M=\frac{C\times 21.48\times D}{V}$	7	$Lbs.=\frac{C\times D^2}{7460\times V}$	8	$\$=\frac{C\times D^2\times K}{746000\times V}$
		4	$M=\frac{N\times D\times 16000}{E\times a\times V}$	5	$Lbs.=\frac{N\times D^2}{10\times E\times a\times V}$	6	$\$=\frac{N\times D^2\times K}{1000\times E\times a\times V}$
	For Minimum Total Initial Cost of Plant.		$M=1226\times C$		$Lbs.=.00766\times D\times C$		$\$=.0000766\times D\times C\times K$
		9	$M=\frac{1,016,000\times N}{E}$	10	$Lbs.=\frac{6.3492\times D\times N}{E}$		$\$=\frac{.063492\times D\times N\times K}{E}$
				11	$Lbs.=362.81\frac{(81-l)}{l}\times N$		$\$=3\ 6281\frac{(.81-l)}{l}\times N\times K$
				12	$Lbs.=362.81\times\frac{\%}{1-\%}\times N$		$\$=3.6281\times\frac{\%}{1-\%}\times N\times K$
				CAPACITY IN ELECTRICAL H. P.		COST.	
Generator...	For Mechanical Horse Power delivered by Motor Shaft.			14	El. H. P. $=\frac{N\times b}{l}$	15	$\$=\frac{N\times b\times G}{l}$
				13	El. H. P. $=\frac{N}{a(1-\%)}$		$\$=\frac{N\times G}{a(1-\%)}$

NOTE.—The small numbers in the left hand upper corners of some columns coincide with the numbers of these formulæ in the test.

TABLE XXVII.

DEFINITIONS OF PRACTICAL ELECTRICAL UNITS.

Quantities to be Measured.	Synonyms.	Symbol	Name of Practical Unit.	Comparative Values.	REMARKS. Fundamental or absolute or C. G. S Units are: Centimeter (C) for Length. Gramme (G) for Mass. Second (S) for Time.
Current.	Strength. Intensity. Rate of Flow. Coulomb per Sec. Volume (obsolete)	C.	Ampere.	Coulombs ÷ Seconds Volts ÷ Ohms.	One Ampere deposits .00032959 gramme, or .005084 grain of copper per second on the plate of a copper voltameter.
Quantity.	Ampere-Second.	Q	Coulomb.	Amperes × Seconds	One hour=3,600 seconds; hence one ampere-hour=3,600 ampere-seconds, or =3,600 Coulombs.
Electromotive Force. Difference of Potential.	**Pressure. Tension.**	**E. M. F. or E**	**Volt.**	Amperes × Ohms. Joules ÷ Coulombs.	One volt=.933 standard Daniell cell, (Zinc sulphate of density 1.4 and copper sulphate of density 1.1)

Resistance		**R.**	**Ohm.**	**Volts÷Amperes.**	**One *legal* Ohm is the resistance of a column of pure mercury, 1 square millimeter in section and 106 centimeters long, at 0° centigrade. 1 *true* Ohm=1.00283 *legal* Ohms**
Capacity.		**K.**	**Farad.**	**Coulombs÷Volts.**	**The Microfarad, one millionth of a Farad, has been generally adopted as a practical unit; the Farad is too large a unit for practical use.**
Power. Activity.	**Electrical H. P Rate of Doing Work Effect Work÷Time.**	**P. or Pw or H. P**	**Watt (Volt-ampere.)**	**Volts×Amperes. Amperes)²×Ohms. (Volts)²÷Ohms. Joules÷Seconds.**	**One Watt=$\frac{1}{746}$ electrical horse power. One electrical H. P.=$\frac{\text{Volts}\times\text{Amperes.}}{746}$ One electrical H.P.=$\frac{(\text{Amperes})^2\times\text{Ohms}}{746}$ One electrical H. P.=$\frac{(\text{Volts})^2}{746\ \text{Ohms.}}$**
Work. Heat Energy.	**Power×Time.**	**W or Wj**	**Joule (Volt-coulomb.)**	**Watts×Seconds. Volts×Coulombs. (Amperes)² ×Ohms ×Seconds. (Volts)²×seconds÷ Ohms.**	**One Joule is the work done or heat generated by a Watt in a second. One Joule is the heat necessary to raise .238 gramme of water 1° C.; or one Joule= 238 caloric or therm. One Joule=.7373 foot pound in a second.**

CHAPTER XVI.

Data Relating to Water Power.

There are numerous natural water powers in the United States running to waste. These water powers may be improved, utilized for electric power transmission, and be made to pay handsomely on the capital invested.

As the improvement of a water power is attended with considerable expense, the amount of power at various seasons should be exactly determined to prevent later disappointments.

The determination of the amount of power is best left to a reliable hydraulic engineer. Any manufacturer of water wheels, however, will send on application complete catalogues giving practical rules and tables for determining the horse power of water powers.

As one cubic foot of water weighs 62.5 lbs., this figure multiplied by the number of cubic feet per minute multiplied by the head in feet, gives the foot-pounds; this divided by 33000 gives the horse power.

Or expressed in a formula:

$$\text{Horse power} = \frac{62.5 \times \text{cubic feet} \times \text{head}}{33{,}000}$$

Very often the quantity of water flowing per minute is not expressed in cubic feet but in "miners' inches." A "miner's inch" of water is approximately equal to a supply of 1.5 cubic feet per minute. (See note at end of chapter).

The quantity of water per minute in a small stream is best measured by means of a weir or dam placed across the stream. The weir tables contained in most of the water wheel catalogues give at once the quantity of water in cubic feet per minute passing over the weir. For larger streams it is necessary to first find the area of cross section in square feet, and then the mean velocity.

The cross sectional area is found in the following manner:

Divide the width of the stream from A to B (see Fig. 20) in any number of equal parts. Find the depth of the stream at each point of division, add all the depths together and divide the result by the number of divisions; this will give the "mean depth;" this multiplied by the width A B gives the cross-sectional area in square feet.

Supposing the measurements are as follows

Depth	at	point	1 — 2	feet
"	"	"	2 — 4	"
"	"	"	3 — 5	"
"	"	"	4 — 5	"
"	"	"	5 — 6	"
"	"	"	6 — 7	"
"	"	"	7 — 7	"
"	"	"	8 — 6	"
"	"	"	9 — 4	"
			Total — 46	feet

Divide 46 by 10, the number of divisions, and 4.6 feet is the mean depth.

Supposing the width A B to be 80 feet, $4.6 \times 80 = 368$ square feet, the areal cross section of the stream.

The velocity of the stream is best estimated by throwing floating bodies on the surface. These "floats" should be

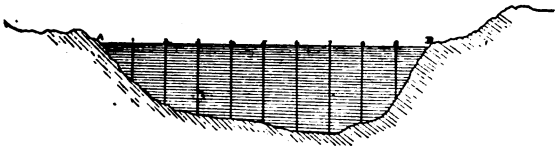


FIG. 20.—CROSS SECTIONAL AREA OF RIVER BED.

of almost the same specific gravity as the water, be weighted on one end and, sink well into the water. They should be started near the middle of the stream where the water has the greatest velocity, and the time should then be accurately measured during which the float will pass a given distance marked by stakes.

Two observers are necessary.

Observers O and P (see Fig. 21) have aligned themselves over stakes a, b, and c, d, respectively.

Observer O is provided with a stop watch (as used for horse races), which he starts when the float passes line a, b; then he turns his face toward P, who gives an optic signal when the float passes over line c, d. O, of course, immediately stops the watch.

The distances a, c and b, d, must be alike.

Supposing this distance is 400 feet, and the time which elapsed while the float passed between a, b, and c, d, is 2 minutes 31 seconds; speed per minute $= \frac{400 \times 60}{151} = 159$ feet.

This, of course, is the velocity near the center and surface of the stream; the velocity near the sides and the bottom, however, is lower. From experiments it has been found that the "mean velocity" is 83 per cent. of the maximum surface velocity.

We must therefore multiply $159 \times .83 = 132$ feet, which is the mean velocity per minute.

If we now multiply this value with 368 representing the cross section in square feet, we get the number of cubic feet per minute; $368 \times 132 = 48576$ cubic feet.

The "head" should be ascertained at high and low water.

It is well to remember that the number of horse power thus ascertained is the *theoretical* energy expended by the water, but only a part of this can be utilized to drive the

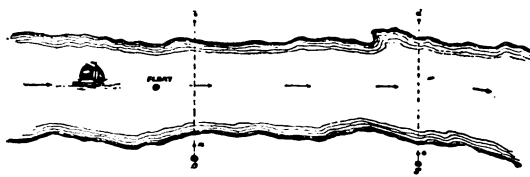


FIG. 21.—MEASURING VELOCITY OF STREAM.

electric generators. The commercial efficiency of the best water wheel is hardly over 80%, besides there is a loss by friction (water in pipes, shafts in bearings, etc.), so that at best not more than 75% of the energy of the water falls can be utilized for driving the electric generators.

A MINER'S INCH.

Dr. T. O'C. Sloane writes in the *Scientific American:*

"For the ampere a peculiarly close analogy is found in a very well known water measurement unit, namely, the miner's inch. The miner's inch is defined as the quantity of water which will flow through an aperture an inch square in a board two inches thick, under a head of water six inches. Here, as in the case of the ampere, we have no reference to any abstract quantity, such as gallons or pounds. There is no reference to time. It is simply and purely a rate of flow, exactly what the ampere is conceived to be in electricity.

"In the illustration, Fig. 22, a representation of a tank whence water is flowing through a hole one inch square extending through a two inch plank, and under a head of water

six inches, is shown. The perforated plank is shown as horizontal, simplifying the pressure question. Referring to these conditions, we may consider the head of water of six inches, as the representative of electrical pressure; in this case representing one volt. The aperture restricting the flow of water may be assumed to represent the resistance of one ohm; the flow through a resistance of one ohm under the pressure of one volt is, of course, one ampere; the flow through the resistance of a one-inch hole two

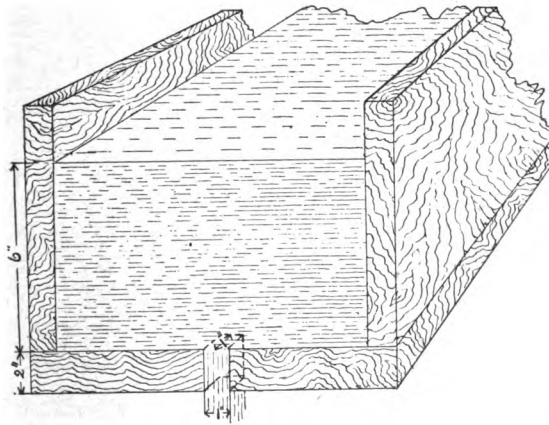


FIG. 22.—A MINER'S INCH.

inches long, under the pressure of six inches to the upper edge of the opening, is one miner's inch.

"The expression, one miner's inch per second would be just as meaningless, or at least redundant, as the expression, one ampere per second. On the other hand, the miner's inch-second is the correct analogue to the ampere-second, the one denotes a specific quantity of water, 0.194 gallon; the other a specific quantity of electricity, a coulomb; 0.194 gallon per second of flow represents a miner's inch; one coulomb per second of flow represents one ampere; 1.94 gallons per second is supplied by ten miners' inches; ten coulombs per second is supplied by ten amperes.

"If we attempt to apply ohm's law to the miner's inch we naturally fail because the laws of hydraulics differ from those of electricity, but, none the less, it is a very excellent analogy."